AN EXPERIMENT IN THE CONSTRUCTION OF MODELS

FOR

THE ANALYSES OF SUSPENSION BRIDGES

WITH

A STUDY

OF

THEIR APPLICATION TO THE DEFLECTION THEORY OF SUSPENSION BRIDGE DESIGN

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Philadelphia, Pa.

March 31st, 1935.

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INTRODUCTION AND GENERAL EXPLANATION.

The purpose of the experiment, outlined in this paper, is to make, by the use of a model of the type hereinafter illustrated and described, a systematic investigation of the action, under various conditions of loading and temperature changes, of a symmetrical three span suspension bridge with unsuspended side spans, and to show that, by this means, results can be obtained in the analysis of the stresses in such a bridge, that are comparable to those obtained by the conventional deflection theory of suspension bridge design.

As far as the writer knows, where experimental analysis has been applied to suspension bridges by the use of models, it has been the practice to obtain, by measuring deformations, certain stresses in the cables and the trusses, and to measure the deflection of the trusses under certain conditions of loading at a few points. The data established by such experiments seem to have been used only as a general check against the results obtained by the deflection theory. These prove that this theory offers a mathematical basis of analysis in which the actual working conditions are closely approximated. However, the deflection theory, in its application, becomes cumbersome and its calculations are long and tedious, as compared to similar requirements for other structural calculations.

On the basis of the data established by this experiment, and its mathematical and dynamic interpretation, there is proposed for consideration, an alternate or supplementary method of analysis, by use of a model of the type illustrated, by means of which the values of certain terms in the fundamental formula for the conventional deflection theory are experimentally determined.

It is assumed that the reader is familiar with the conventional deflection theory of suspension bridge analysis, as the limitation of time and space prevent the presentation of same. The fundamental formula of the deflection theory is $M=M'-(H+Hw)n-H(y)$.

M = The actual bending moment at a given point in the stiffening truss.

M' = Simple beam bending moment at same point without the cable assisting.

Hw = Horizontal component of cable stress due to dead load.

H = Horizontal component of cable stress due to live load.

y = Ordinate to cable parabola under dead load at point considered. (Referred to axis GG see Figures I and II).

n = Deflection of stiffening truss at point considered.

In the above equation M' and Hw are easily calculated. All the other quantities consume much time in calculation, but if a model of the type illustrated is set up and any loading is applied, it requires only a short time to measure these values from this model by the process outlined in the following pages. When once determined, values of M follow quickly and shears if desired by finding values of M at adjacent suspension points. This is a brief summary of what has been done, and what is to be outlined and illustrated in this paper. It will be readily seen, by those familiar with the application of the conventional deflection theory, the amount of time and labor saved once the model is set up and placed in operation.

This method and procedure is not proposed as a check on the conventional deflection theory, which, as previously stated, has been carefully studied and confirmed by experimental analysis, but it parallels it step by step and arrives at results which for all practical purposes will be shown to be in close agreement. This

permits this method of model analysis to be used either independently in design or investigation or as an independent check on the mathematical computations of the conventional deflection theory.

The model is so built that it bears a certain geometrical and dynamic ratio to the bridge that is to be investigated and constructed. In other words, the conditions of mathematical similitude are satisfied.

The fact is emphasized that it has been the objective to build a model that is simple and that could be set up in a minimum amount of time. No effort was made to be precise, in the sense of the physicist, but every care was taken to see that the model would simulate, in its vital parts, the full size original to a high degree of accuracy. Every effort was made to evolve a device that would be commercially possible for the consulting engineer and possess the necessary accuracy to obtain results that from the standpoint of design or investigation would be sufficient.

The bridge here considered has many of the properties of the one figured in "Modern Framed Structures", Part II, by Johnson, Bryan and Turneaure. The main exception is that herein, for purposes of simplification, the side spans have been considered as unsuspended. Cable sizes, stiffening truss properties, loadings, and center span geometrical properties are exactly the same.

The model as finally constructed proved to be quite simple, but certain problems arose during its evolution which offered considerable difficulty. These will be pointed out and commented upon in their proper places.

In regard to the time required for such an analysis the reader will observe that the work contained herein is no criterion of what would be required for the analysis of a particular structure.

In actual practice, there would be fewer loading conditions to be considered, whereas here nearly every possible condition likely to occur has been dealt with in order to obtain a full set of deflection curves. These curves permit a visualization of the deflection actions under the various conditions of loading. The investigation of a particular bridge of the type shown could be made by two men in two or three weeks, including the time of building the model, provided they followed carefully the method outlined herein in order to avoid the delays due to difficulties that invariably arise in the development of a new mechanism. It is not intended to give the impression that there are hard and fast rules for the construction of such a model, other than those of mathematical similitude, but there are certain things that must not be done, many of which can only be developed by trial. A model in itself is a product of evolution and generally can be improved upon. Adjustments of the model, especially for dead load, may seem difficult, but in reality they offer no trouble, for once being made they are fixed throughout the experiment and from then on there is what may be very properly called a "locked" system.

For quantitative comparison eight cases have been analyzed which amply illustrate the principles and methods involved. The results determined by model analysis for four conditions of loading with two of temperature for each condition have been tabulated and opposite each case results determined by the conventional deflection theory have also been tabulated. The calculations are given in complete detail for those based on the data established from model. The results figured by the conventional deflection theory for the eight cases are given, but the calculations themselves, owing to the limitation of space, are not given but can be furnished to anyone who is interested in the details of this study.

Qualitatively, a set of deflection curves covering every condition of moving uniform load of a critical nature that is likely to occur have been constructed. The development of these curves is considered one of the most important features of the experiment, as they give a complete visual demonstration of just what happens to a stiffening truss under load. To a certain degree they are a "yard-stick" for the deflections of all stiffening trusses for bridges having the same sag ratio. Other comparisons have been made and they will be commented upon later.

A summary of the purposes of this experiment is as follows:

1. To show that it is practicable to construct a model sufficiently accurate for all practical purposes, comparatively inexpensive, simple and adaptable with slight modifications to all ordinary types of suspension bridges, that will show and record the action of the bridge under various conditions of loading and temperature changes.

2, To show that with the data so established the stresses throughout the bridge can be easily determined with results that are in close agreement with those obtained by the conventional deflection theory.

The term "conventional deflection theory" or "deflection theory" is used in this paper to describe the generally accepted deflection theory, based entirely on mathematical analysis. This theory is set forth in any standard work on the subject and has been fully elaborated and commented upon in the March, 1934, Proceedings of the American Society of Civil Engineers.

The term "model" is used in this paper not in the strict sense of a miniature reproduction of the structure to be investigated, but to describe a mechanism similar to the one illustrated,

which while possessing the proper geometrical and dynamic ratios, is designed primarily for the purpose of registering and measuring the action of the applied forces under various conditions of loading.

The model described was set up and the results taken in the Geodetic Laboratory of the University of Pennsylvania, where the proper facilities for space, accuracy of observation, etc. existed, and is at the present time available for inspection.

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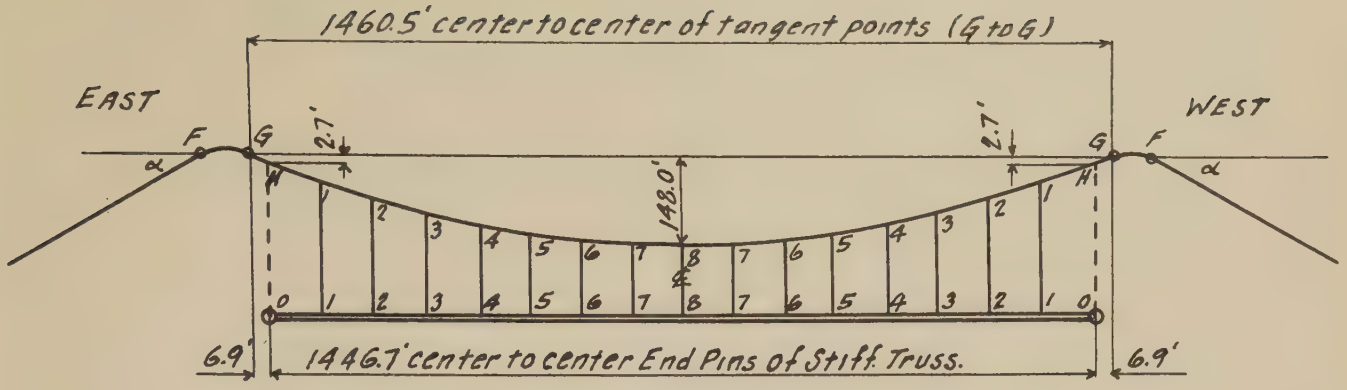


Figure I. (Dimensions of Field Bridge).

Figure I shows the center span of the bridge to be investigated and referred to hereinafter as the field bridge. For purposes of this investigation sixteen panels have been assumed.

Side spans are not shown but have the following dimensions:

Length of cable side span = 872.41'
Secant α = 1.03
Horizontal projection of side span cable = 847.00'

Figure II, page 8, shows the dimensions to which the model was constructed in accordance with the geometric scale used and the dimensions as determined below.

Geometric scale of model to field bridge, 1 to 192.

$\frac{1}{16}$" measured on model = 1' measured on field bridge.

1" measured on model = 16' = 192" measured on field bridge.

Span of model G to G = $\frac{1460.5 \times 12}{192}$ = 91.28125"

Sag of model cable at center = $\frac{148 \times 12}{192}$ = 9.25"

Vertical distance GH of model = $\frac{2.7 \times 12}{192}$ = .16875"

Horizontal distance GH of model = $\frac{6.9 \times 12}{192}$ = .43125"

c.to c. end pins of stiffening truss of model = 91.28125-2(.43125) = 90.41875"

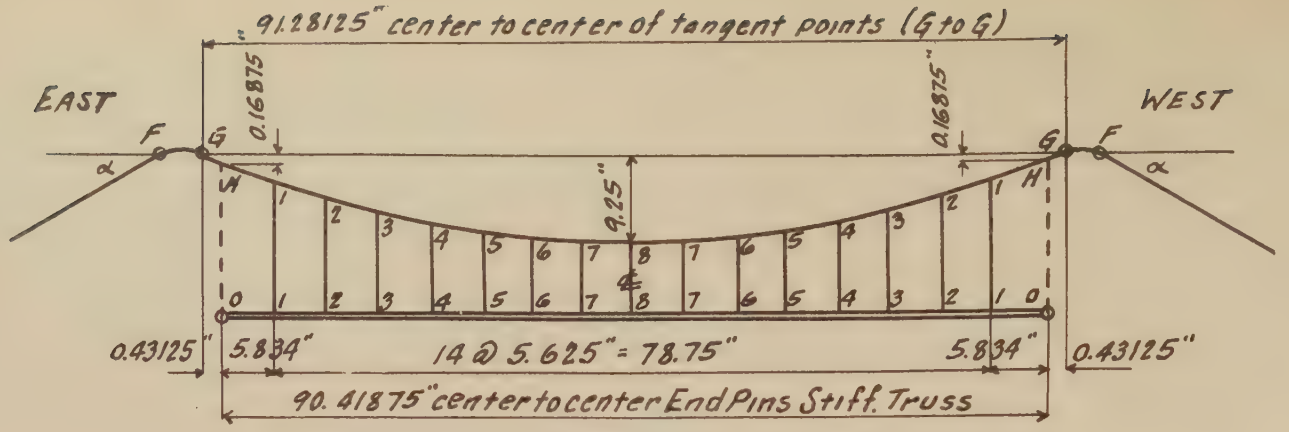


Figure II. (Dimensions of Model).

The ordinates of the cable parabola from a horizontal line at the low point are given in Table I below. To these are added 3" for the details and 3" for length of wire for center hanger giving the lengths of all hangers as listed. (See Figure III on page 25). The figures are for the length of hangers under full dead load.

TABLE I.
LENGTHS OF CABLE HANGERS.

Hanger Number	Ordinate to Parabola from Horizontal Line Thru Low Point	Constant See Fig. III	Length of Hanger Center of Cable to Center Stiffening Truss
8-8	0.0000	6.0000	6.00
7-7	0.1405	6.0000	6.14
6-6	0.5620	6.0000	6.56
5-5	1.2645	6.0000	7.26
4-4	2.2480	6.0000	8.25
3-3	3.5125	6.0000	9.51
2-2	5.0580	6.0000	11.06
1-1	6.8845	6.0000	12.88
0-0	Special Double Wooden Strut		

The length of cable G to G is calculated by the following approximate formula:

$$L = \text{Length of cable G to G} = \ell\left(1+\frac{8}{3}r^2-\frac{32}{5}r^4\right)$$

ℓ = Horizontal distance G to G = 91.28125"
f = Sag of cable = 9.25"
$r = \frac{f}{\ell} = .1013$

$\therefore L = 93.7176"$

The "free" cable length can be obtained by shortening L by the dead load stretch or final adjustment can be made by setting center point for correct dead load sag and adjusting elevation of each panel point of stiffening truss by instrument observation on panel point scale.

End panel lengths of model were made slightly different from other panels so as to give workable dimensions for laying off panel lengths from center.

With the geometric outline of the model established the dynamic ratios are determined as follows. Refer to Figure V, page 26, showing the complete set up of the model as finally built.

Nomenclature and Properties.

Df = Deflection of the field bridge.

Wfs = Total live load on the field bridge = 4000#/ft.x 1446.7 ft.

Efs = Modulus of Elasticity of stiffening truss of the field bridge = 29,000,000#/sq."

ℓfs = Span of the field bridge stiffening truss = 1446.7'

If = Moment of Inertia of the field bridge stiffening truss = 6,321,600 in^4.

Af = Area of the field bridge cable = 275.0 sq."

Lfc = Length of the field bridge cable (G to G).

Efc = Modulus of Elasticity of the field bridge cable = 29,000,000#/sq."

Pfc = Average stress in the field bridge cable under full live load i.e. (Wfs on the stiffening truss).

efc = Stretch of the field bridge cable due to Pfc.

W'fs = Total dead load on the field bridge = 5,820#/ft.x 1446.7 ft.

Dm = Deflection of the model.

Wms = Total live load required on the model to simulate the same total live load condition on the field bridge.

Ems = Modulus of Elasticity of the model stiffening truss = 1,900,000#/sq.". A stiffening truss made of a single piece of wood was used. The Modulus of Elasticity of this wood was determined in advance.

ℓms = Span of the model stiffening truss = 90.41875"

Im = Required Moment of Inertia of the model stiffening truss.

Am = Area of model cable used = .000661 sq.".

Lmc = Length of the model cable G to G under dead load = 93.7176"

Emc = Modulus of Elasticity of the model cable. (A high strength piano wire .029" in diameter with a Modulus of Elasticity of 29,000,000#/sq." was used).

Pmc = Average stress in the model cable under full live load. (Wms on the model stiffening truss).

emc = Stretch of the model cable due to Pmc.

W'ms = Required dead load on the model.

K = A deflection constant depending on form of loading.

C = The geometric scale constant = $\frac{\text{Size of Field Bridge}}{\text{Size of Model}} = 192$

For Wms of model proceed as follows:

$$efc = \frac{PfcLfc}{AfEfc} \qquad emc = \frac{PmcLmc}{AmEmc}$$

$$efc = Cemc \qquad Lfc = CLmc$$

$$\frac{efc}{Lfc} = \frac{Cemc}{CLmc} \qquad \therefore \frac{efc}{Lfc} = \frac{emc}{Lmc}$$

Substituting $\frac{Pfc}{AfEfc} = \frac{Pmc}{AmEmc}$ or $\frac{Pfc}{Pmc} = \frac{AfEfc}{AmEmc}$

But $\frac{Pfc}{Pmc} = \frac{Wfs}{Wms}$

Giving $\frac{Wfs}{Wms} = \frac{AfEfc}{AmEmc}$

$$\therefore Wms = \frac{(AmEmc)}{(AfEfc)} Wfs \qquad (1a).$$

As the Modulus of Elasticity of the field bridge cable is the same as the Modulus of Elasticity of the model cable

$$Emc = Efc$$

$$\therefore Wms = \frac{Am}{Af} \times Wfs \qquad (1b).$$

For required Im of model

$$Df = K \frac{Wfs\,\ell^3 fs}{Efs\ If} \qquad Dm = K \frac{Wms\,\ell^3 ms}{Ems\ Im}$$

$$Df = CDm \qquad CK \frac{Wms\,\ell^3 ms}{EmsIm} = K \frac{Wfs\ C^3\ell^3 ms}{EfsIf}$$
$$\ell fs = C\ell ms$$

$$\therefore Im = \frac{(If)}{(C)^2}\frac{(Wms)}{(Wfs)}\frac{(Efs)}{(Ems)} \qquad (2a).$$

Substituting in (2a) the value of Wms given by (1a)

$$Im = \frac{(Efs)}{(Ems)}\frac{(Am)}{(Af)}\frac{(Emc)}{(Efc)}\frac{(If)}{(C^2)} \qquad (2b)$$

But Emc = Efc

$$\therefore Im = \frac{(Efs)}{(Ems)}\frac{(Am)}{(Af)}\frac{(If)}{(C^2)} \qquad (2c)$$

Equations (1a) and (2a) = (2b) give Wms and Im where the Modulus of Elasticity of the field bridge cable is different from that of the model cable and likewise where the Modulus of Elasticity of the field bridge stiffening truss is different from that of the model stiffening truss. They cover the most general cases.

Equations (1b) and (2c) give Wms and Im where the Modulus of Elasticity of the field bridge cable and model cable is the same but where the Modulus of Elasticity of the field bridge stiffening truss is different from that of the model stiffening truss. This is the case that is being considered and the equations to be used for Wms and Im are

$$Wms = \frac{Am}{Af} \times Wfs \qquad (1b)$$

$$Im = \frac{(Efs)(Am)(If)}{(Ems)(Af)(_{C}2)} \qquad (2c)$$

Calculation for Total Live Load on the Model.

Am = .000661 sq."

Af = 275 sq."

Wfs = 4000 x 1446.7

Efs = 29,000,000#/sq."

Ems = 1,900,000#/sq."

If = 6,321,600 in^4

C = 192 C^2 = 36,864

Substituting in equation (1b)

$Wms = \frac{.000661}{275} \times 4000 \times 1446.7 = 13.91\#$ = The total live load required on the model to simulate the total live load on the field bridge.

Calculation for Size of the Model Stiffening Truss.

Substituting in equation (2c)

$Im = \frac{29,000,000}{1,900,000} \times \frac{.000661}{275} \times \frac{6,321,600}{36,864} = .00629126 = \frac{bd^3}{12}$

The width of the model stiffening truss is 1" (b = 1")

$d^3 = .07549512$

d = .4226" which = $\frac{27}{64}$"

The piece of wood used for the model stiffening truss was planed and then sandpapered down to $\frac{27}{64}$" in depth.

Calculation for Dead Load Required on the Model.

Wms = 13.91#

W'ms = Total dead load required on the model.

Wfs = Total live load on the field bridge.

W'fs = Total dead load on the field bridge.

$$\frac{W'fs}{Wfs} = \frac{5,820 \times 1446.7}{4000 \times 1446.7} = \frac{5.82}{4} = 1.455$$

$$\frac{W'ms}{Wms} = \frac{W'fs}{Wfs} = 1.455$$

Therefore W'ms = 1.455 Wms = 13.91 x 1.455 = 20.24#

The total dead load required on the model = 20.24#

Application of Live Load to Model.

16 cardboard boxes were provided to carry the live load on model as shown in Figure V, page 26.

1 ounce = 28.3495 grams.

$\frac{13.91 \times 16 \times 28.3495}{16}$ = 394 grams per live load box including weight of box.

Application of Dead Load to Model.

The weight of the stiffening truss, hangers, cable and fittings was found to be 1863 grams. This is called the fixed dead load. The full required dead load as previously determined is 20.24#

20.24 x 16 x 28.3495 = 9181 grams = the total required dead load.

1863 grams = the fixed dead load.

7318 grams = required superimposed dead load.

To carry the superimposed dead load 16 cardboard buckets were provided and suspended as shown in Figures IV and V.

The load per bucket including weight of bucket and fittings was $\frac{7318}{16}$ = 457.38 grams.

With the data determined, as outlined in the previous pages, the main supports, pulleys, blocks, etc., were assembled as shown in Figure V. The length of the model cable G to G was then laid off on the cable wire, the suspenders fastened at their approximate locations, connected to the stiffening truss, and the model bridge swung. The proper superimposed dead load was then placed in the dead load buckets and adjustments made for correct center elevation and correct suspender lengths under full dead load.

The suspenders at the start were made as near to their correct theoretical length as practicable, and with the details shown in Figure III, permitting adjustment of length, further and final adjustments were later made. The lower ends of suspenders were located horizontally by means of the holes for same accurately spaced in the stiffening truss. By dropping a plumb bob from the main steel supporting angles to the holes for connecting suspenders to stiffening truss the correct vertical alignments for suspenders were determined. By means of the adjustable detail at the top of suspenders (see Figure III), the suspenders were brought into proper alignment with plumb bob lines and to their correct position. A more refined adjustment of the suspenders for length was made later, using a large pair of "jaw dividers". In doing this care was taken to maintain enough weight in the dead load counterweight bucket to keep the stiffening truss from undergoing any serious distortions.

With the suspenders properly adjusted, the correct sag of the cable at the center was obtained by varying the dead load counterweight, and measurements taken from a fine steel wire stretched taut across the centers of the supporting pulleys. With the correct center sag fixed the versine corresponding to each suspender was checked and fixed, measuring from this horizontal wire, all as shown

in Figure V.

It was found that these adjustments as outlined above had to be repeated several times in order to reduce the variation from the theoretical figures to negligible quantities. This type of model with the cable extending over a pulley and leading to a counterweight bucket readily permits of many of these adjustments.

When the dead load adjustment was complete, the wooden struts at the ends connecting to the stiffening truss at bottom and wood block in the supporting angle at the top, were placed in position and holes bored for pins to suit the correct adjusted position of the stiffening truss.

Attention is called to the fact that the design of the model has been made so that the cable wire has no sharp bends. On the counterweight end the wire cable was discontinued and a very flexible beaded chain connected to it by means of a pulley, and passed over the counterweight pulley so that there were no sharp bends in the cable wire. The necessity of avoiding sharp bends was developed by experiment when the first set up showed that the bending of the cable wire around the pulley leading to the counterweight would give serious error in the results.

The west end of the cable was held by a friction clamp as shown in Figures V and IX. Wherever the wire cable was discontinued several turns of it were taken around brass pulleys to avoid breaks which might occur due to bending back of wire

With the model correctly adjusted for dead load condition the weight in the dead load counterweight bucket was carefully determined and recorded. It remained untouched throughout the remainder of the experiment, and no further dead load adjustments of any kind were required.

The dead load tension in the model cable measured by the weight in the dead load counterweight bucket was found to be 12,300 grams.

For the remainder of the experiment only the live load counterweight bucket required load adjustment at counterweight, except for the case where temperature effect with no live load was considered.

The first observation was on the tension in the model cable from dead load, this giving a general check on the accuracy of the model. The observed tension in the model cable under full dead load as measured by the weight required in the dead load counterweight bucket was found to be 12,300 grams.

By means of formula (4b), whose development is indicated on page 23, the horizontal component of the cable of the field bridge under full dead load is calculated.

Having $Hw^{\#} = T'mc$ (in grams) x J' (in inches) x 137.15 (4b)

where

T'mc = Tension in model cable in grams = 12,300 grams

J' = 6.2653"

$Hw^{\#}$ = Horizontal component of the field bridge cable, under full dead load, in pounds.

$Hw^{\#} = 12,300 \times 6.2653 \times 137.15 = 10,569,000\#$ (From Model)

Working analytically $Hw^{\#}$ is found to be

$$Hw^{\#} = \frac{wl^2}{8f} = \frac{5,820 \times (1446.7)^2}{8 \times 145.3} = 10,480,000\# \quad \text{(Analytically)}$$

From the above it is seen that the model gives an excess of 0.85%. The above difference of less than one percent is caused by three facts. First, parabolic conditions were assumed in the analytical calculations, whereas actually the cable is an equilibrium polygon. Second, the dead load adjustment of the model cannot be made perfect. Third, the model cable (as does the field bridge cable) has a bending resistance which is always neglected in theoretical computations.

It will be observed, that for dead load, no adjustment of the model cable is necessary for any unsuspended side span cable or backstay stretch. This is due to the fact that the center span is brought to its correct geometrical position and figure by the weight added to the dead load counterweight bucket, which operation in itself gives effect to the backstay stretch just as the tipping back of the towers of the field bridge after erection and their swinging over into position after the application of the dead load gives effect to the backstay stretch in bringing the field bridge to the correct dead load position.

Investigation of Live Load Conditions.

With the model adjusted for full dead load condition the elevation of all panel points is established and recorded from instrument observation on panel point scales. A bench mark is established on the stiffening truss at the west end for reference in establishing elevations of panel points with respect to same, and a separate bench mark away from the model is established to keep a check on any movement of the model as a whole. The transit is set on the horizontal scale H.S.

For the assumed live load condition to be investigated the corresponding live loads are placed on the model. Upon the application of the live load with no change in temperature the H.S. scale moves as the result of the live load alone. Its movement is neutralized by placing weight in the live load counterweight bucket. After the H.S. scale is brought back to zero position the weight in the live load counterweight bucket represents the tension in the model cable at the east end tangent point due to this live load alone. The elevations of the various panel points are then taken and recorded. This operation is repeated for each live load condition to be investigated.

The horizontal component H of the cable tension is required and is obtained by determining the angle that the cable makes with the horizontal in the east end panel. This angle is obtained by hanging light weight scales on the cable in the east end panel and measuring the same directly, or it is obtained mathematically by the use of formula (5) given below. (See page 24 for the development of the formula).

$H = T \times J \times 137.15$ (4a) See page 24.

T = Tension in the model cable (in grams) from live load.

$J = \sqrt{39.2540 - 4.731\,(d-k)-(d-k)^2}$ (in inches) (5)

d = deflection of model truss at panel point #1 (in inches).

$k = .3532\,\Delta c$ (in inches).

Δc = Movement of cable over supporting pulley, (in inches), due to side span cable stretch and temperature change considering all spans.

$\Delta c = 0$ Where side spans are not considered and no temperature change considered.

This procedure up to this point has been made on the assumption of inelastic backstays, i.e. for purpose of analysis a single span suspension bridge.

The effect of the unsuspended side span cable or backstay stretch in the model investigation is provided for in the following way. On application of any particular live load to the field bridge the backstays stretch a certain amount in accordance with the stresses set up in the same by these loads. This stretch can be readily calculated and also the amount of the horizontal movement it produces in the top of the tower, (the tower assumed as pinned at bottom). The amount of this horizontal movement at top of the tower is then converted into its model equivalent and is given its effect in the deflections of the model stiffening truss by running the model cable over the supporting pulleys an amount that will produce this

horizontal model equivalent distance. (The pulleys in this operation are the model equivalents of towers with fixed tops and a cable sufficiently flexible to move over the tower saddles instead of the tops of the towers moving). The effect of the two backstays, one at each end, is provided for by running twice the amount over the east pulley. The calculations for this procedure are as follows:

Effective length of backstay of field bridge = 872.41 (See Figure I).
$\text{Sec}\alpha$ = 1.03 (See Figure I).
$H^{\#} = T \times J \times 137.15$ (4a) (See page 24).
J = 6.2653" (See Figure II)
(d and k in formula (5) assumed equal to zero as the dead load horizontal distance is sufficiently close to J for this particular calculation).
T = Tension from live load or weight in live load counterweight bucket in grams.
To establish a basis for figuring other conditions by proportion assume T = 1000 grams.
By equation (4a) H = 859,286 lbs. or 859,286 lbs. is the horizontal component of the stress in the field bridge backstay caused by a tension in the cable at top of tower whose model equivalent is 1000 grams.
This 859,286 lbs. causes a stretch in one backstay of

$$\frac{859,286 \times 872.14 \times 12}{275 \times 29,000,000} \times 1.03 = 1.1614"$$

Reducing this to the resulting horizontal motion at top of tower 1.1614 x 1.03 = 1.1962" or a total motion for two backstays of 2.3924".
Reducing this to its model equivalent

$$\frac{2.3924"}{192} = .01246"$$

For converting this horizontal figure into its corresponding figure for correction of model cable length a true parabolic condition is assumed for center span cable.
Let center span = ℓ
Length of cable center span = L
$\Delta\ell$ = change in length ℓ
ΔL = change in length L
Deduced from formula for length of parabola

$$\Delta L = \Delta\ell\left(1-\frac{8r^2}{3}+\frac{96r4}{5}\right)$$

$\Delta\ell$ = .01246" as determined previously

$$r = \frac{148}{1460.5} = .1013 = \frac{9.25}{91.28125}$$

$$\Delta L = .9746572\ (\Delta\ell) \qquad (6)$$
$$= .0122"$$

Hence for a live load tension in the model cable of T = 1000 grams the length of model cable must be altered by .0122" to provide for backstay effect. For any other value of T the length of the model cable is altered proportionately.

This alteration in the length of the model cable is effected by running the cable over the east pulley by the required amount as shown by transit observation on the horizontal scale H.S. This observation on the H.S. scale gives a horizontal figure but as the slope of the cable at this point is very flat it was found that this gave practically the same results as measuring the required amount of alteration along the cable at this point.

For any particular live load condition to be investigated, with the loads applied, the weight in the live load bucket after equilibrium has been established, less the weight to be taken out to obtain the necessary alteration in cable length, to provide for backstay stretch, gives the required T for this condition of loading. With equilibrium finally established for these conditions, instrument readings are made and elevations of all panel points established and recorded. With T determined the value of H is figured from the formula H = T x J x 137.15 (4a). Hw was determined as shown on page 18. From the record of elevations of panel points of model stiffening truss, the deflection n of any panel point of the field bridge is determined by model proportion. This establishes all the terms and permits the direct application of the fundamental formula of the deflection theory

i.e. $M = M' - (H+Hw)n - H(y)$.

Also from the recorded elevation of the panel points of the model stiffening truss, by model proportion, the deflections of all panel points of the field bridge stiffening truss are determined, and from these the elastic curve of the field bridge stiffening truss can be plotted for each condition of loading.

Summary of Adjustments for Backstay Stretch Effects.

For every 1000 grams occurring in live load bucket of counterweight, increase length of model cable by .0122" to care for backstay stretch.

Temperature Effects.

These were handled by calculating the amount of change in cable length due to change in temperature and accomplished in a way similar to that previously given for backstay stretch for live load. The procedure and calculations are as follows:

Assumed change in temperature plus 53° F.
Coefficient of linear expansion = .0000065
For backstays (see Figure I). Horizontal projection of backstay = 847'

$$847' \times .0000065 \times 53 = .29184'$$

Effect of two backstays = .29184 x 2 = .58368'
Model equivalent in inches = $\frac{.58368}{192} \times 12 = .03648"$ (horizontally)
Reducing this to its equivalent length on the curve of the model cable see formula (6), page 20.

$$\Delta L = .9746572 \times .03648 = .0356"$$

This is the required length of model cable that must be run over the east pulley to give the proper model effect for a change of temperature of +53° F. in the field bridge backstays.

For temperature effect in the center span
Length of center span cable = 1520' (includes portion over towers)

$$1520' \times .0000065 \times 53 = .52364'$$

Model equivalent = $\frac{.52364 \times 12}{192} = .03273"$

This is the required length of model cable that must be run over the east pulley to give the proper model effect for a change of temperature of +53° F. in the field bridge center span cable.

After the model cable has been run over the east pulley the required amounts by taking weight from the live load counterweight bucket and proper equilibrium has been established, this weight taken out of the live load counterweight bucket, represents the tension or the model equivalent of the field bridge cable tension, produced by a change of temperature of +53° F. This permits analysis for all temperature effects, and also permits the elastic curve of the field bridge stiffening truss, showing temperature effect combined with any live load condition, to be plotted.

Summary of Adjustments for Temperature Effects for +53° F.

For side spans increase length of model cable	.0356"
For center span increase length of model cable	.0327"
For total effect increase length of model cable	.0683"

Development of Formulae for Finding H and Hw from Tension in Model Cable

Refer to nomenclature given on pages 9 and 10 with the following additional notations:

T'fc = Dead load tension in the field cable in panel 0-1 east.
Tfc = Live load tension in the field cable in panel 0-1 east.
T'mc = Dead load tension in the model cable in panel 0-1 east.
Tmc = Live load tension in the model cable in panel 0-1 east.
Hw = Horizontal component of T'fc.
H = Horizontal component of Tfc in pounds.
J = Horizontal projection of the distance 6.6970" which is the length of the model cable from the east tangent point to the first hanger, neglecting small changes in this length due to cable stress conditions from live load. J will vary with each condition of live loading. On page 19 it has been shown how to obtain J experimentally. On page 24 a formula is given for J. The two methods, for all practical purposes, are in close agreement.
J' = The value J assumes under dead load condition = 6.2653".
N = A constant for each condition of live load.
N' = A constant for dead load.
U = Ball bearing pulley friction factor = 1.0015.
R = A constant depending on the length of live load.

From dynamic considerations $\frac{W'ms}{Wms} = \frac{W'fs}{Wfs}$ or $\frac{Wfs}{Wms} = \frac{W'fs}{W'ms}$

The conventional deflection theory makes the assumption that the entire dead load is carried directly by the cable and that the horizontal component set up in the cable due to this dead load always remains the same, i.e. stresses are set up in the stiffening truss only from live load or temperature variations from normal. With this assumption, the live load tensions Tfc and Tmc for the same condition of loading will be proportional to the live load producing them. This leads to the following equations

$Tfc = NRWfs \qquad Tmc = NRWms$

Due to the dynamic and geometric similitude of the model and the field bridge NR affects Tmc and Tfc by the same amount.

But $NR = \frac{Tmc}{Wms}$ $\therefore$ $Tfc = \frac{Tmc}{Wms} Wfs$ or $Tfc = \frac{Wfs}{Wms} Tmc$

using the relation $\frac{Wfs}{Wms} = \frac{W'fs}{W'ms}$

$$\therefore Tfc = \frac{W'fs}{W'ms} \times Tmc$$

or placing in actual numerical values and taking account of the ball bearing pulley friction factor

$$Tfc = \frac{Tmc \times 5{,}820\#/ft. \times 1446.7\ ft.}{20.24\#} \times 1.0015$$

Tmc is measured in grams and H is desired in pounds.

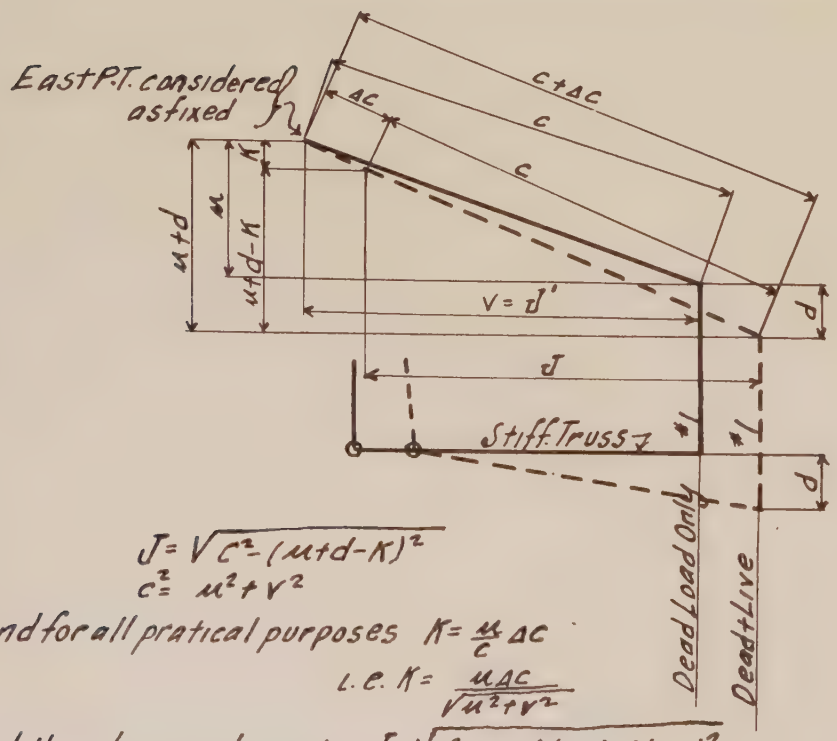
$$\therefore H^{\#} = \frac{Tmc \text{ in grams}}{28.3495 \times 16} \times \frac{J}{6.6970} \times \frac{5820 \times 1446.7}{20.24} \times 1.0015$$

$$\therefore H^{\#} = Tmc\,(\text{in grams}) \times J\,(\text{in inches}) \times 137.15 \quad (4a)$$

In an exactly similar manner

$$Hw^{\#} = T'mc\ (\text{in grams}) \times J'\ (\text{in inches}) \times 137.15 \quad (4b)$$

Development of Formula for J.



Heavy full line represents dead load condition.

Heavy dotted line represents dead plus live load condition.

$$J = \sqrt{C^2 - (u+d-K)^2}$$

$$c^2 = u^2 + v^2$$

and for all pratical purposes $K = \frac{u}{c}\Delta c$

i.e. $K = \frac{u\Delta c}{\sqrt{u^2+v^2}}$

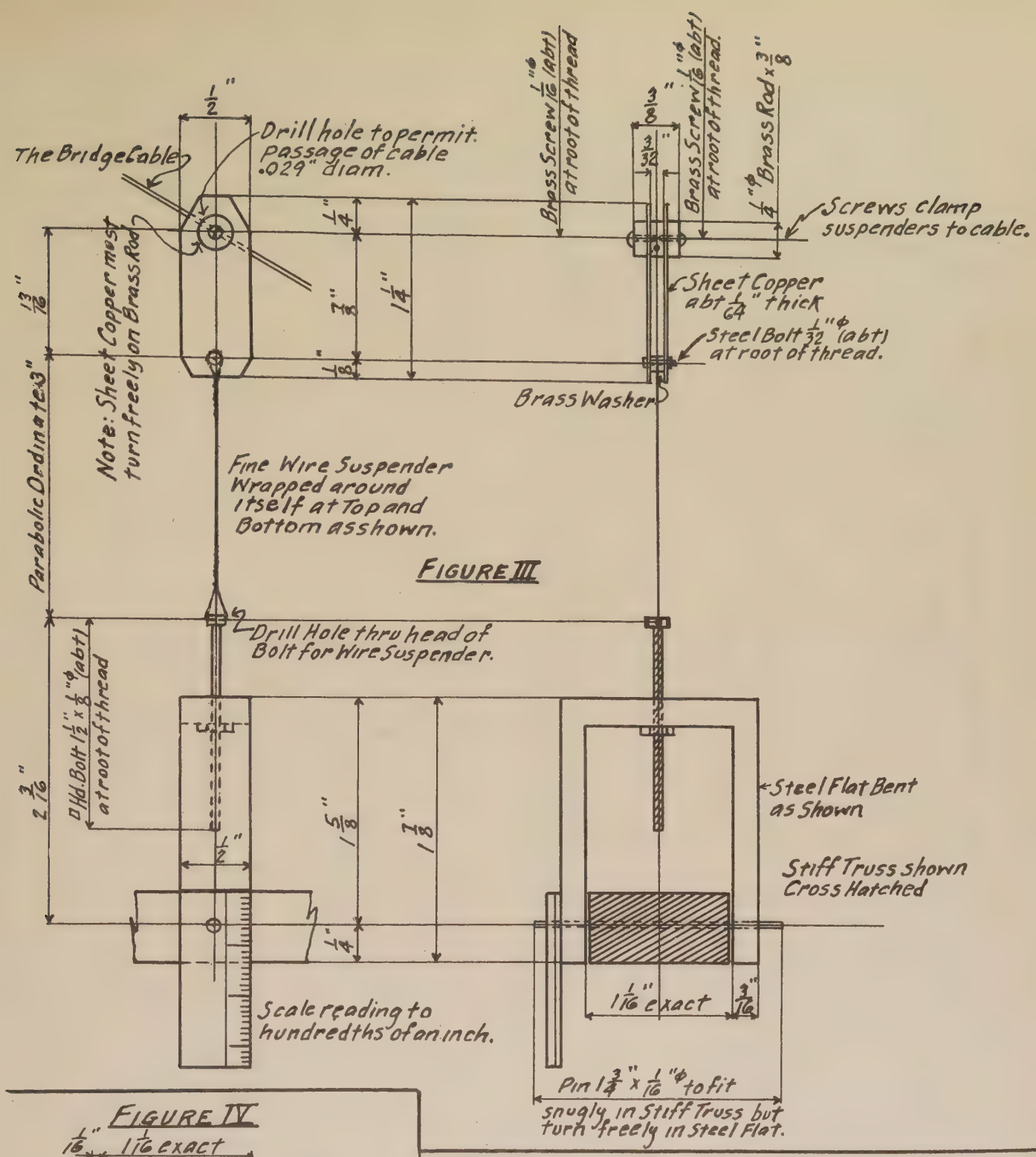
and the above reduces to $J = \sqrt{V^2 - 2u(d-K) - (d-K)^2}$

$u = 9.2500 - 6.8845 = 2.3655$

$V = 5.8340 + .4313 = 6.2653 = J'$

$\therefore K = .3532\Delta C$

$$\therefore J = \sqrt{39.2540 - 4.7310(d-K) - (d-K)^2} \quad (5)$$


The Bridge Cable
Drill hole to permit passage of cable .029" diam.
Brass Screw 1/16" ϕ (abt) at root of thread
Brass Screw 1/16" ϕ (abt) at root of thread.
1/4" ϕ Brass Rod x 3/8"
Screws clamp suspenders to cable.
Sheet Copper abt 1/64" thick
Steel Bolt 1/32" ϕ (abt) at root of thread.
Brass Washer
Note: Sheet Copper must turn freely on Brass Rod
Parabolic Ordinate 3"
Fine Wire Suspender Wrapped around itself at Top and Bottom as shown.
FIGURE III
Drill Hole thru head of Bolt for Wire Suspender.
Hd. Bolt 1 1/2" x 1/4" ϕ (abt) at root of thread
Steel Flat Bent as Shown
Stiff Truss shown Cross Hatched
Scale reading to hundredths of an inch.
1 1/16" exact
Pin 1 3/4" x 1/16" ϕ to fit snugly in Stiff Truss but turn freely in Steel Flat.

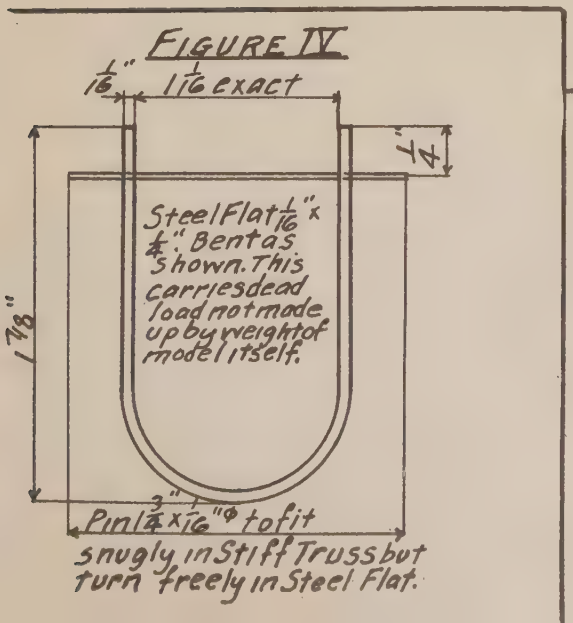


FIGURE IV
1/16"
1 1/16 exact
1/4"
Steel Flat 1/16" x 1/4" Bent as shown. This carries dead load not made up by weight of model itself.
1 7/8"
Pin 1/4" x 1/16" ϕ to fit snugly in Stiff Truss but turn freely in Steel Flat.

To connect the hangers to the cable and stiffening truss the details as shown in Figure III were devised. The stiffening truss was arbitrarily assumed 1" wide. Its depth is determined later on when dynamic ratios are dealt with. Figure IV shows the detail necessary to carry the excess dead load which is occasioned by the insufficient weight of model itself.

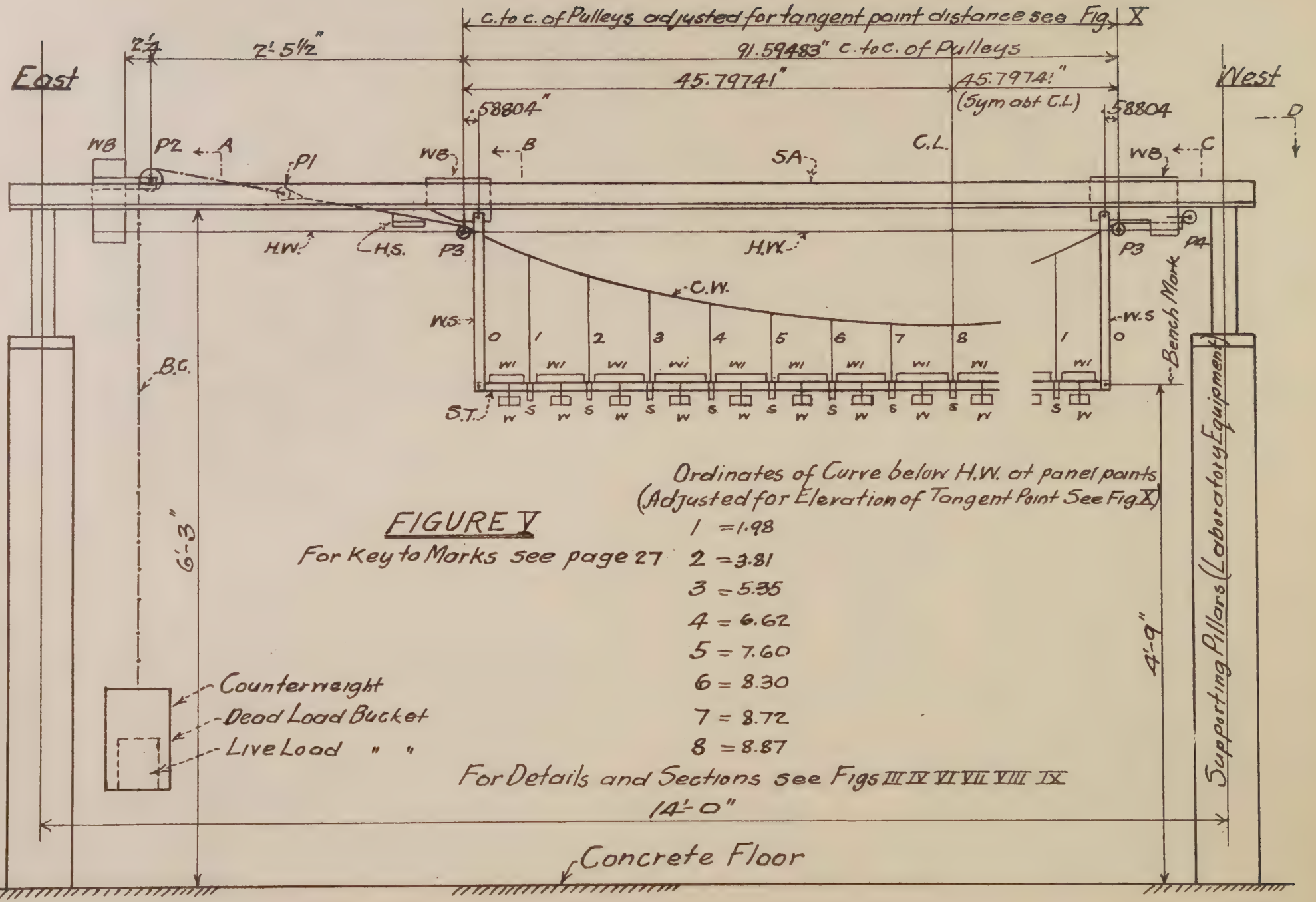


FIGURE V

For Key to Marks see page 27

Ordinates of Curve below H.W. at panel points
(Adjusted for Elevation of Tangent Point See Fig. X)

1 = 1.98
2 = 3.81
3 = 5.35
4 = 6.62
5 = 7.60
6 = 8.30
7 = 8.72
8 = 8.87

For Details and Sections see Figs III IV VI VII VIII IX

Key to Marks on Figure V.

SA - Main Supporting Angles, 2 angles 2 1/2" x 2 1/2" x 1/4", 1" b to b with washers and bolts spaced about 12" c to c. Bolts tightened and adjusted to give clamping effect.

HW - Fine steel wire stretched tight across center of supporting Pulleys, for use in maintaining adjustments and making check on same.

WS - Wooden Struts adjusted and pinned top and bottom after adjustment for dead load condition.

CW - Cable Wire.

BC - Beaded Chain.

P1 - Splice Pulley, Cable to Chain.

P2 - Counterweight Pulley (Ball Bearing).

P3 - Supporting Pulleys.

P4 - Connecting Pulleys.

ST - Stiffening Truss.

WB - Wooden Blocks.

W - Superimposed dead load buckets.

W1 - Live load boxes.

S - Vertical scales for measuring deflection.

HS - Horizontal scale for measuring horizontal movement of cables.

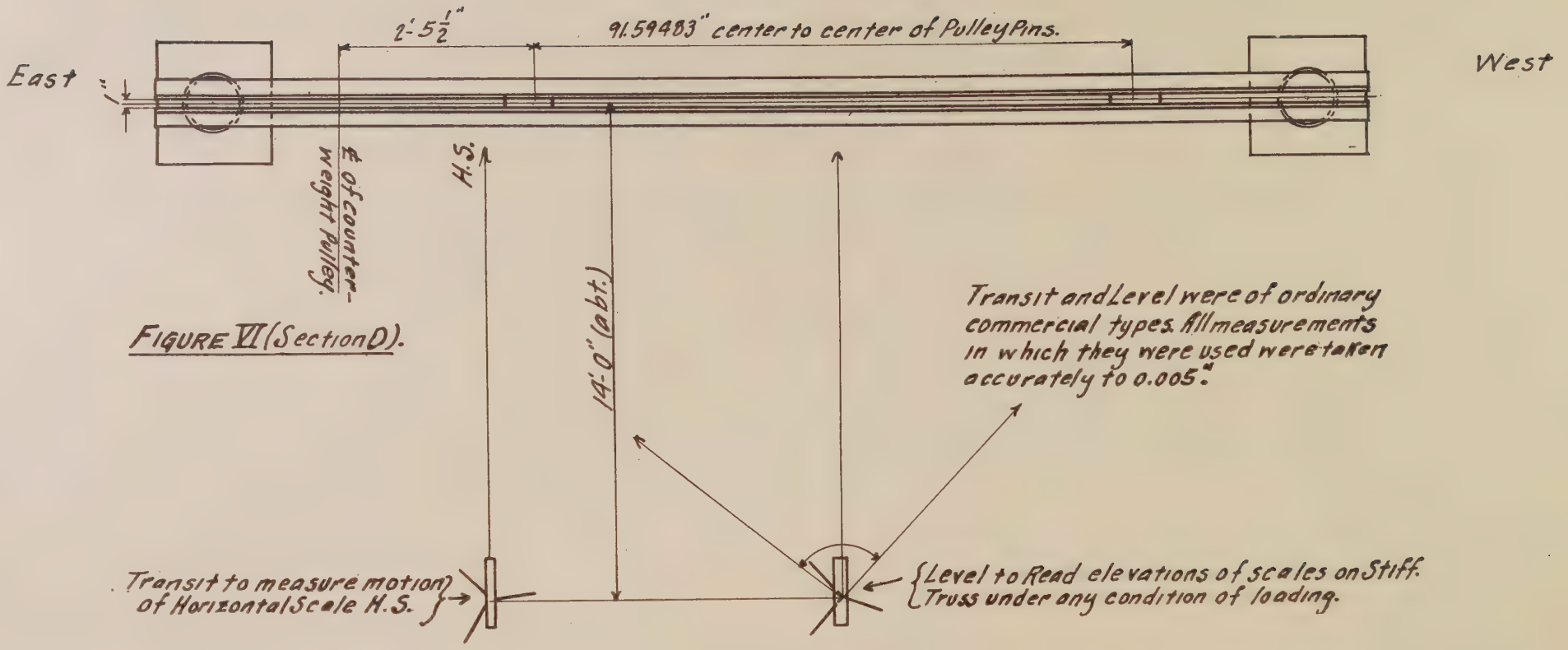

East
West
2'-5½"
91.59483" center to center of Pulley Pins.
℄ of counter-weight Pulley.
H.S.
14'-0" (abt.)
Transit and Level were of ordinary commercial types. All measurements in which they were used were taken accurately to 0.005".
Transit to measure motion of Horizontal Scale H.S.
Level to Read elevations of scales on Stiff. Truss under any condition of loading.

FIGURE VI (Section D).

Figure V is a front elevation of the model. The loads shown are merely illustrative. They were placed from time to time to suit the requirements of each particular live loading condition to be observed.

Figure III, page 25, shows details of the suspender connections to cable and to stiffening truss.

Figure IV, page 25, shows the detail for carrying the superimposed dead load buckets.

Figures VII, VIII, and IX, page 29, show details of the construction.

Figure VI, page 28, is a plan of the model and instrument set up showing the general locations of transit and level for taking readings on the horizontal and vertical scales.

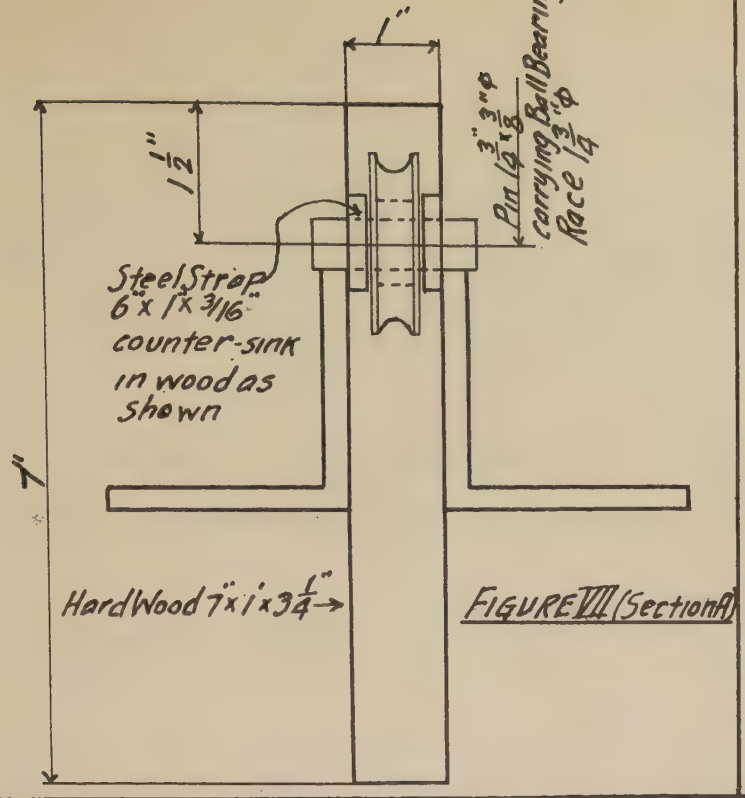


FIGURE VII (Section A)

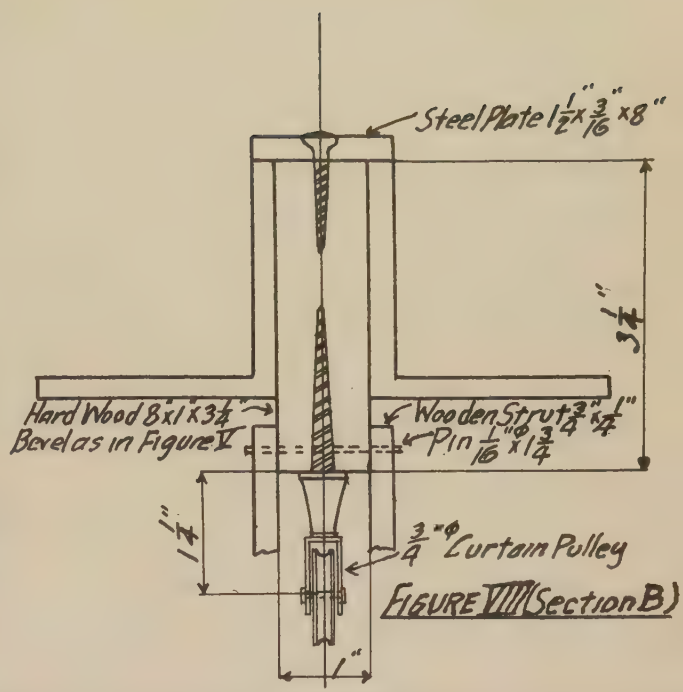
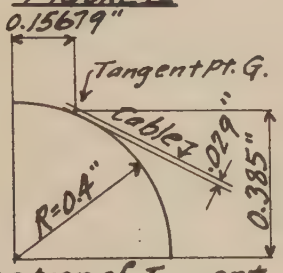

Steel Plate 1 1/2" x 3/16" x 8"
3 1/2"
Hard Wood 8" x 1" x 3 1/4"
Bevel as in Figure V
Wooden Strut 3/4" x 1/4" x 1"
Pin 1/16" Φ x 1 3/4"
1 1/4"
3/4" Φ Curtain Pulley
1"

FIGURE VIII (Section B)

FIGURE X



Location of Tangent Point as shown.
Note: Pulleys overrun .05" in diameter.

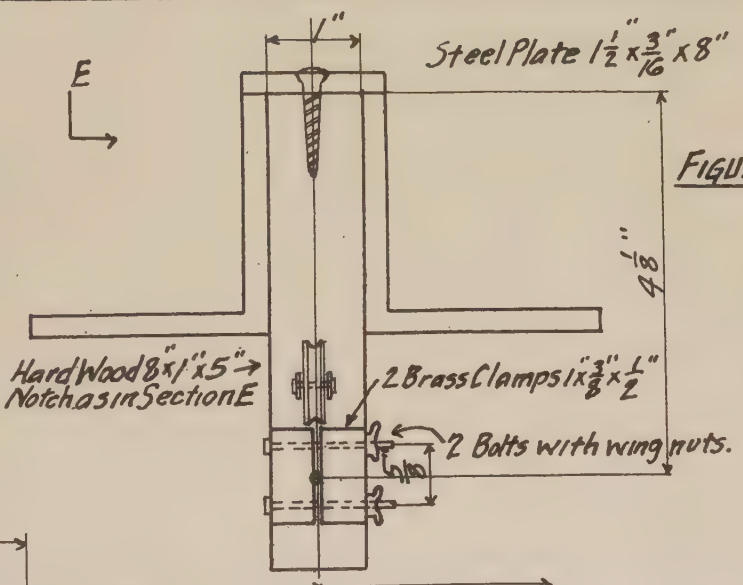
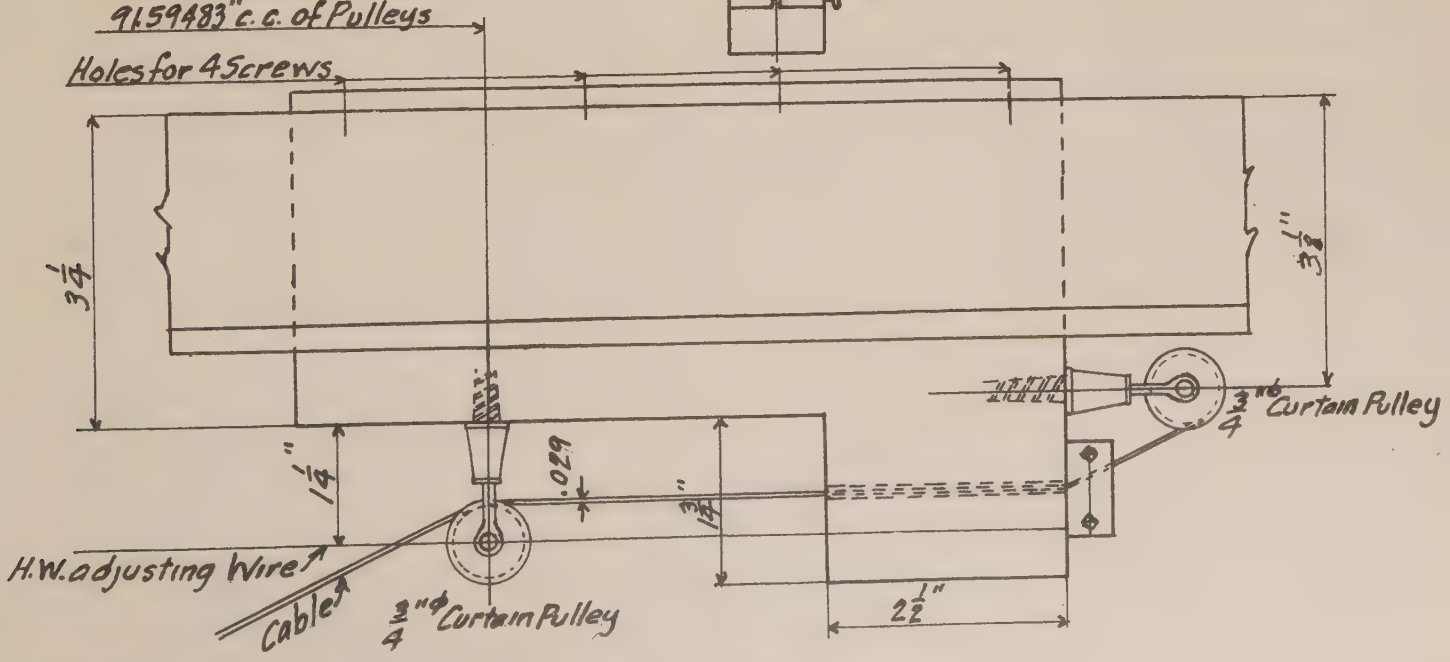


FIGURE IX (Section C)



RESULTS.

Table II shows the different loading conditions investigated partially or otherwise with the assumptions used. For identification of the loading conditions and assumptions, the following system of numbers and letters is used:

The Roman numeral indicates the number of panels loaded. The Arabic number immediately following indicates the number of panels unloaded from the East end.

The last letter following the Roman numeral and the Arabic number indicates the following:

c - the bridge considered as a single span suspension bridge (no backstay stretch) with temperature normal.

d - the bridge considered as a three span suspension bridge with unsuspended side spans with temperature normal.

f - the bridge considered as a three span suspension bridge with unsuspended side spans with temperature 53°F above normal.

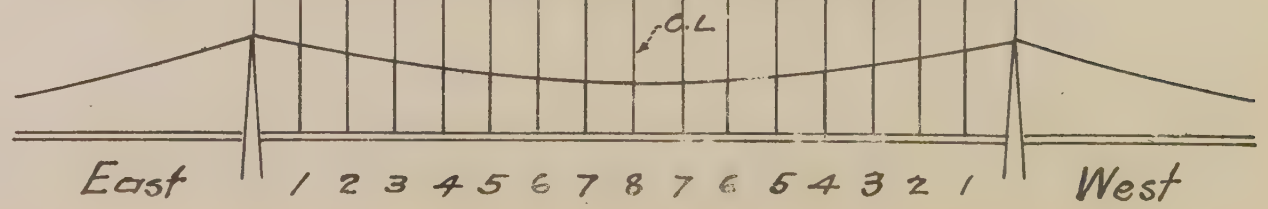
Wherever the letter "g" occurs in the table the elastic curve of the stiffening truss has been plotted.

The following eight loading conditions were selected as a basis of comparison for results obtained by the model analyses, and by the conventional deflection theory. For these the deflections, shears, moments, etc., were established by both methods and the results tabulated for comparison in tables III and IV, pages 60 and 61.

Case V od	Case VII od
Case V of	Case VII of
Case VI 5d	Case XVI od
Case VI 5f	Case XVI of

Table II

Loading		Bridge Condition			Position of Loads
Number of Panels Loaded	Number of Panels unloaded from East End	Centre Span only no backstay stretch C	Unsuspended Side Spans Temperature Normal d	Unsuspended Side Spans Temperature +53°F f	Circles indicate Live Loads and correspond to position of Live Load Boxes on model Numbers are panel point numbers of Centre Span
IV	0	g	g		
"	1	"	"		
"	2	"	"		
"	3	"	"		
"	4	"	"		
"	5	"	"		
"	6	"	"		
V	0	"	"	g	
VI	0	"	"		
"	1	"	"		
"	2	"	"		
"	3	"	"		
"	4	"	"		
"	5	"	"	g	
VII	0	"	"	g	
VIII	0	"	"		
"	1	"	"		
"	2	"	"		
"	3	"	"		
"	4	"	"		
X	0	"	"		
"	1	"	"		
"	2	"	"		
"	3	"	"		
XII	0	"	"		
"	1	"	"		
"	2	"	"		
XIV	0	"	"		
"	1	"	"		
XVI	0	"	"	g	



g-indicates graphs showing deflections have been plotted

Graphs G1 to G9, inclusive, show, for various conditions of loading, the elastic curves of the stiffening truss for a single span suspension bridge (no backstay stretch) with temperature normal.

Graphs G1A to G9A, inclusive, show, for the same conditions of loading, the elastic curves of the stiffening truss for a suspension bridge with unsuspended side spans with the temperature normal. A comparison of these indicates clearly the effect of side spans or backstay stretch on the deflections.

Graph G10 shows the effect of a $+53^{\circ}$ F. increase in temperature on the deflections of the stiffening truss for a suspension bridge with unsuspended side spans and with no live load.

Graphs G11 to G14, inclusive, are self explanatory, and cover the loading conditions and cases figured and tabulated in Tables III and IV.

Graph G15 shows the elastic curves for a single panel live load, moving across the span, for the stiffening truss of a suspension bridge with unsuspended side spans.

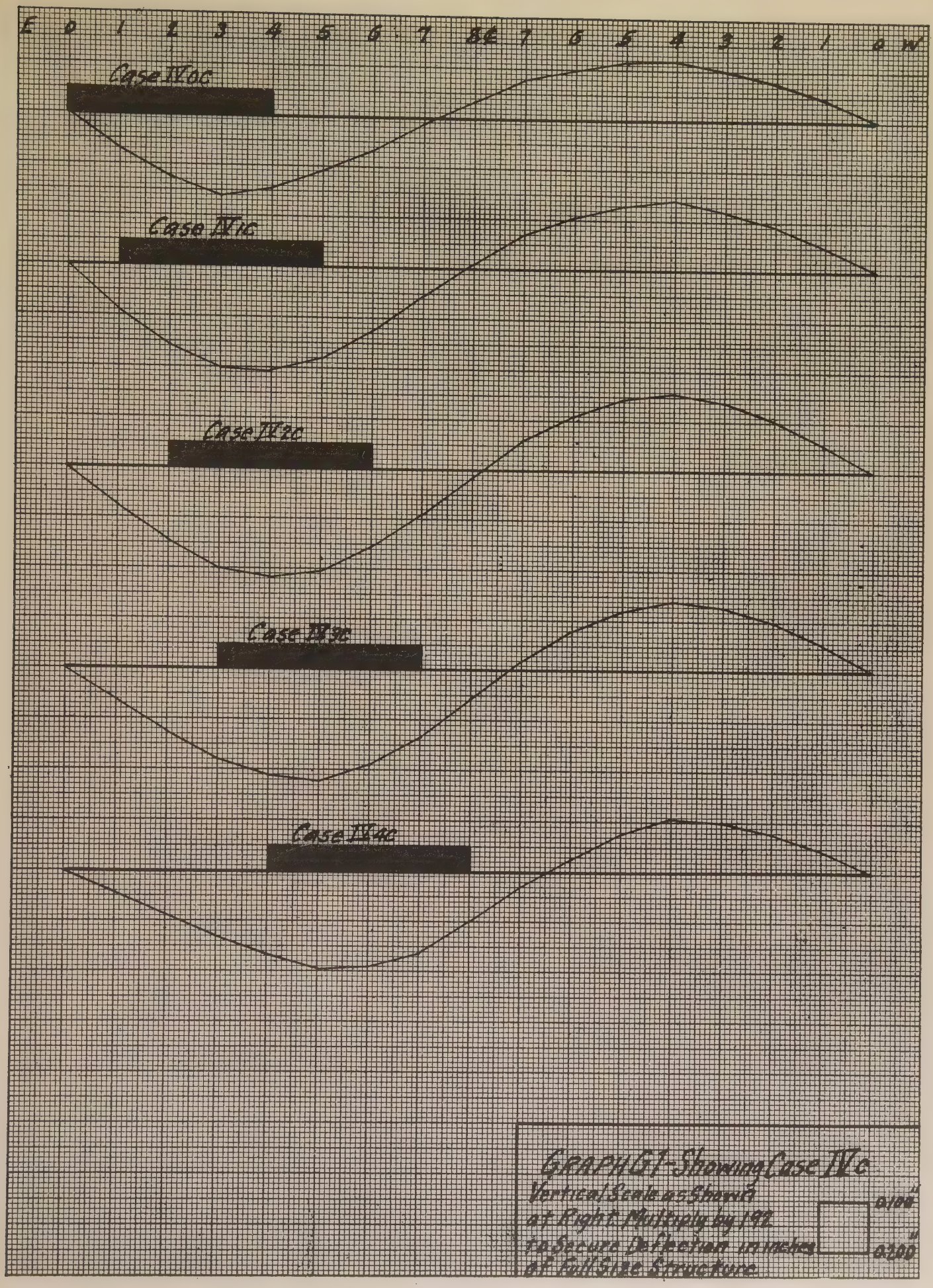
Graph G16 shows the elastic curves for a single panel live load, moving across the span, for the stiffening truss of a single span suspension bridge (no backstay stretch).

Graphs G 17 and G18 show the values of H for various loading conditions for a bridge with unsuspended side spans.

In interpreting these graphs, refer to Table II for loading conditions used and system covered.

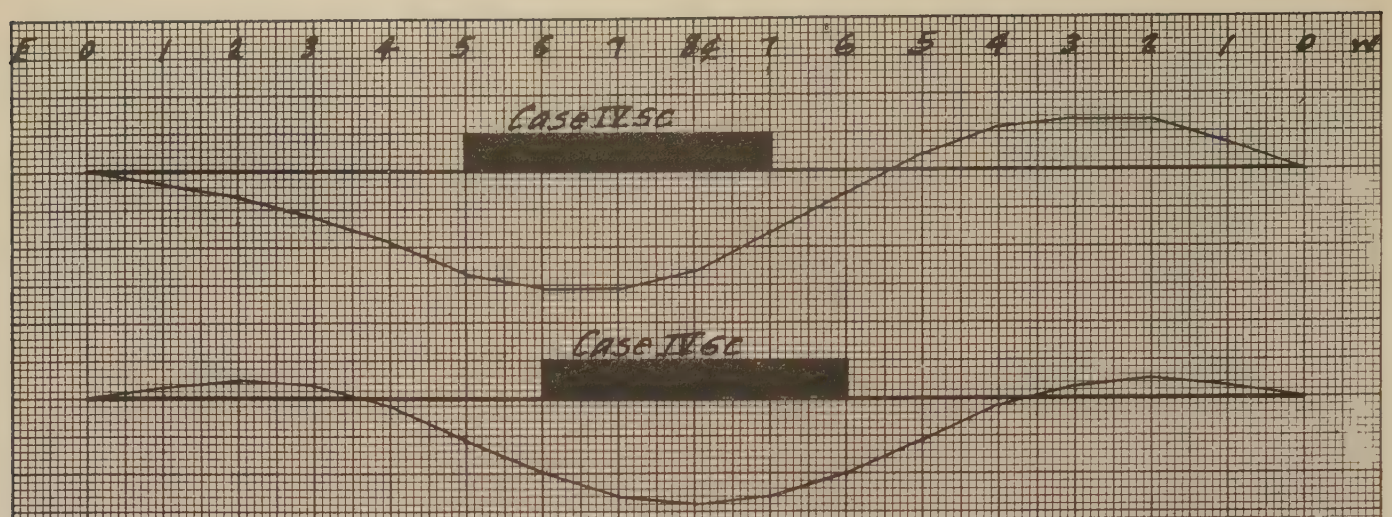
All of these graphs were plotted from data established by observations on the model. Vertical figures taken from these graphs give the recorded deflection of the model in inches. When multiplied by 192 they give the deflections of the field bridge in inches.

Pages 62 and 63, inclusive, show the calculations necessary to convert data read from the model into the proper form for use in the fundamental formula and for comparison with results obtained by the conventional deflection theory. Attention is called to these in contrast to the lengthy computations involved in the use of the conventional deflection theory.



GRAPH G1-Showing Case IVc
Vertical Scale as Shown
at Right Multiply by 192
to Secure Deflection in inches
of Full Size Structure

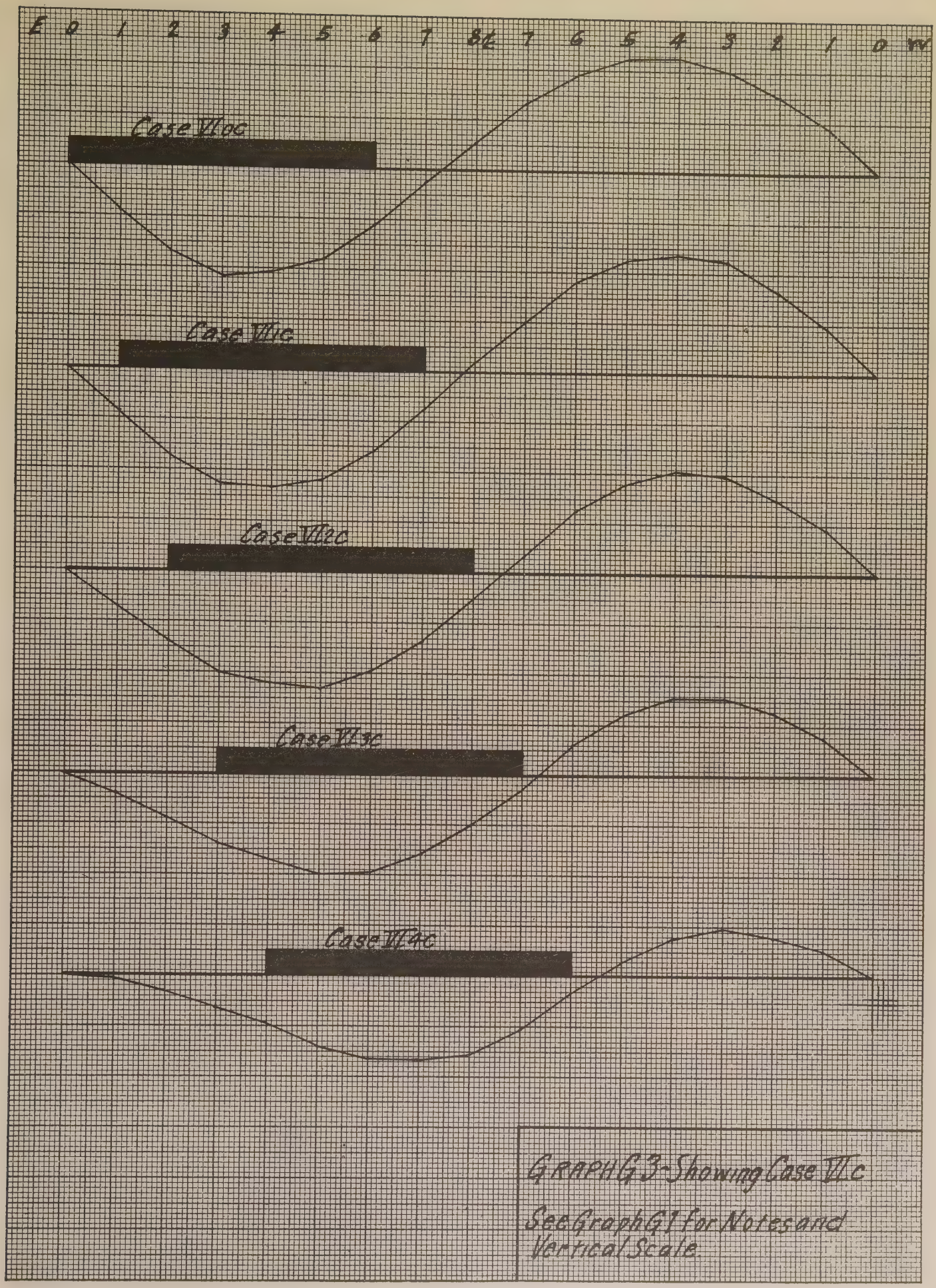
0.100"
0.200"



Case IV 6C represents a condition for live load symmetrically spaced about the center line. There are 15 cases of various live loading conditions symmetrically spaced about the center line similar to this, and one of temperature alone, shown in the following set of graphs. In some of these the deflections measured by the model were unsymmetrical near the east and west ends. The differences in values were very small and for purposes of plotting the average of the two values was used.

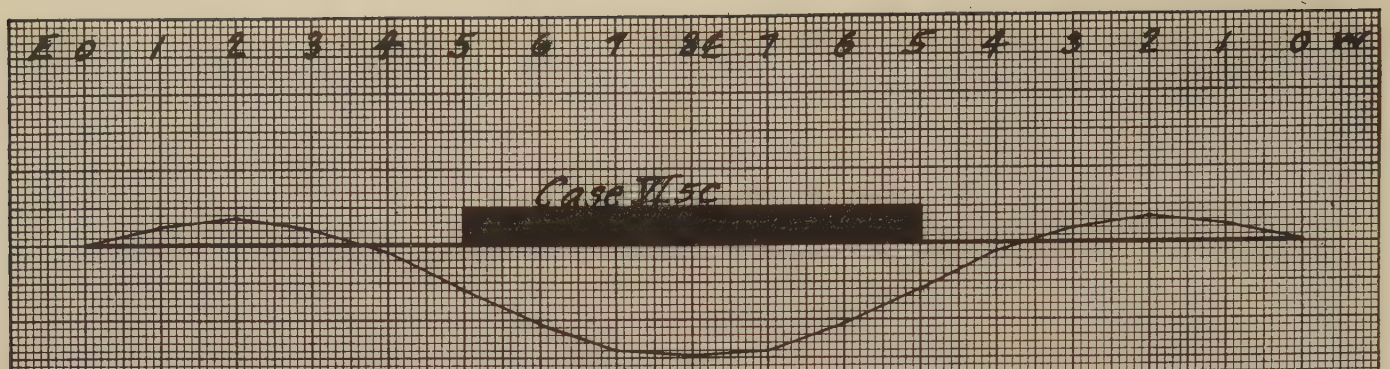
GRAPH 62 - Showing Case IV 6C Cont.

See Graph 61 for Notes and Vertical Scale



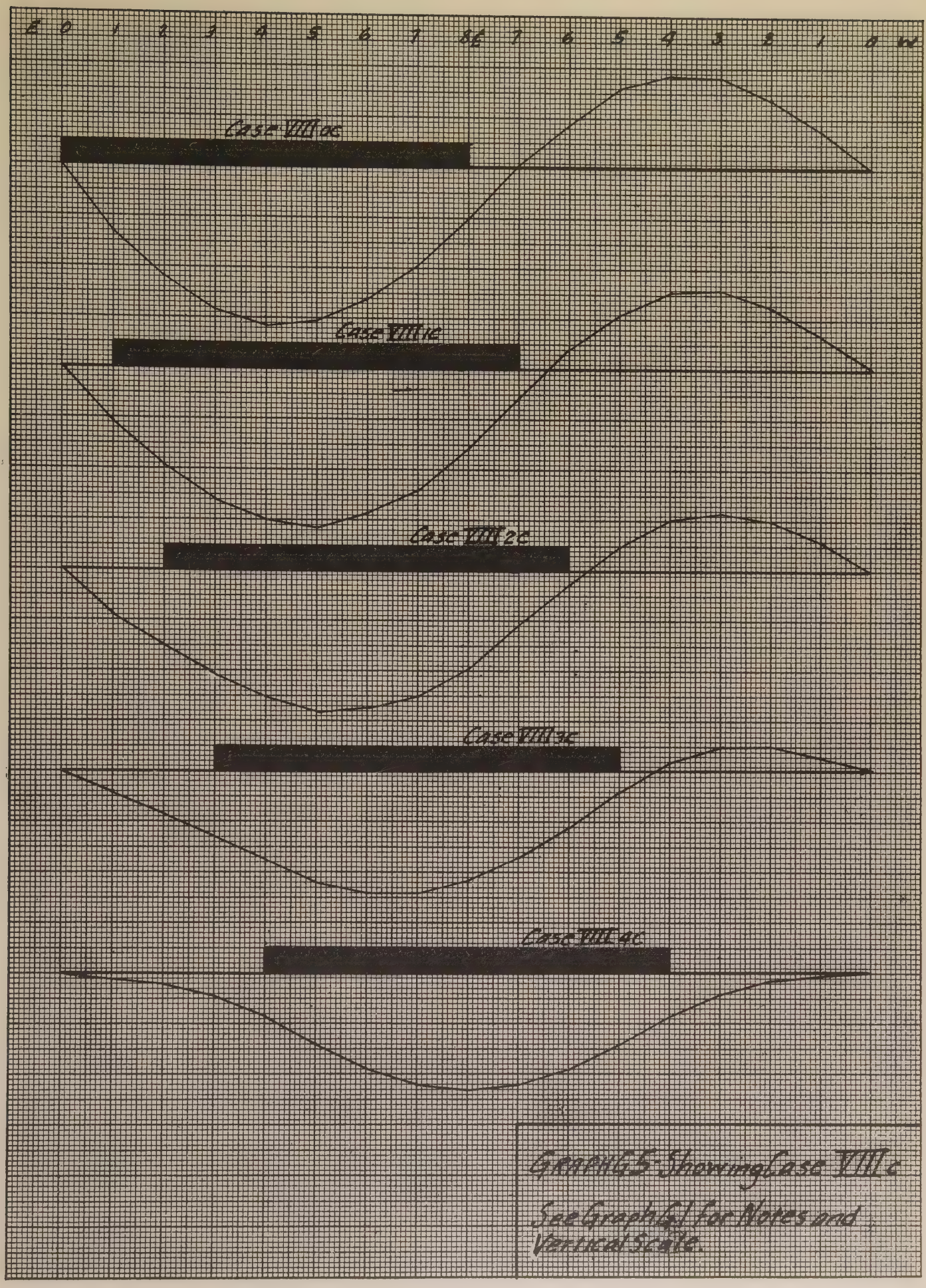
GRAPH G 3 - Showing Case VI c

See Graph G 1 for Notes and Vertical Scale.



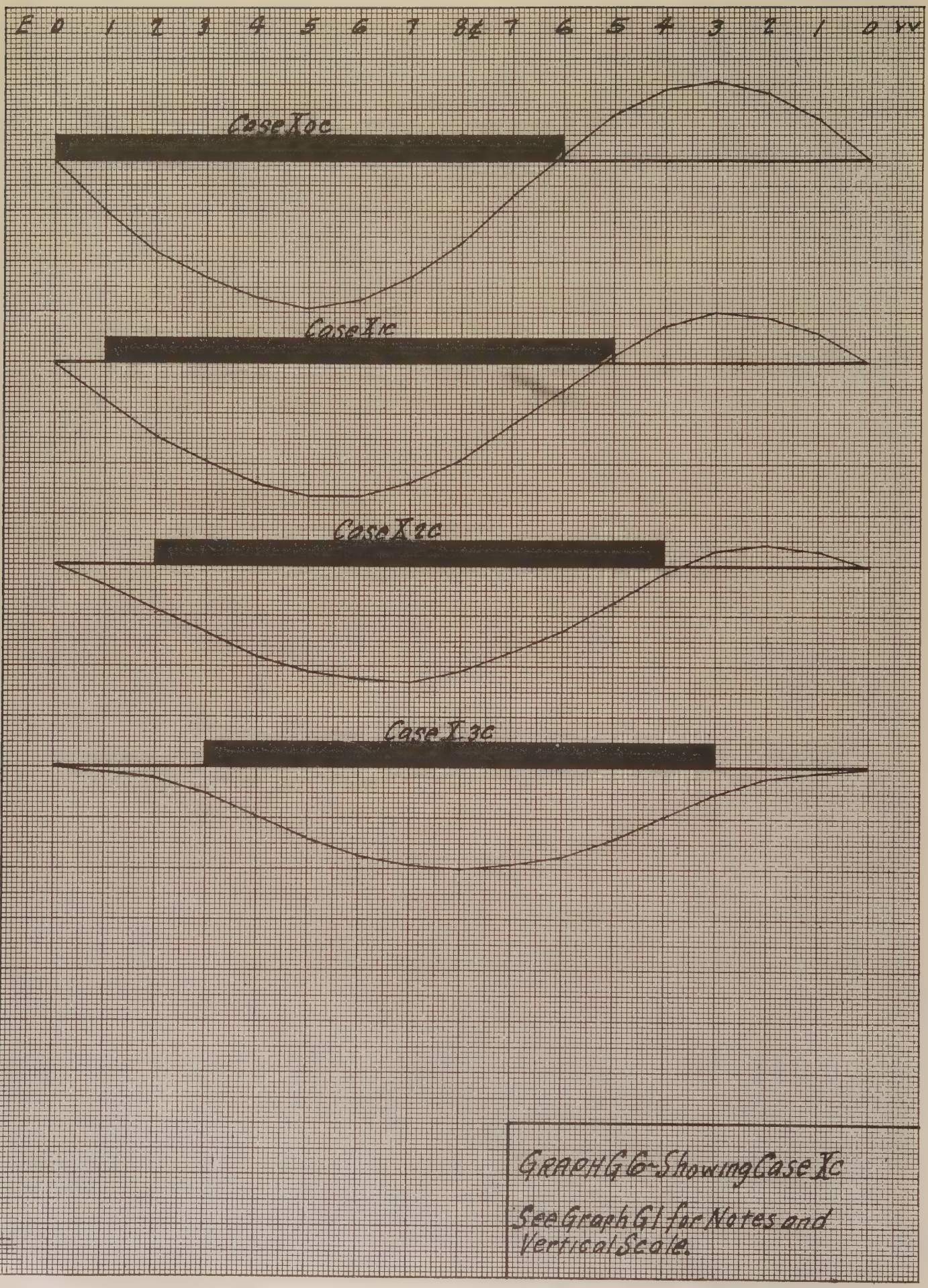
Graph G4-Showing Case VIIc Cont.

See Graph G1 for Notes and Vertical Scale



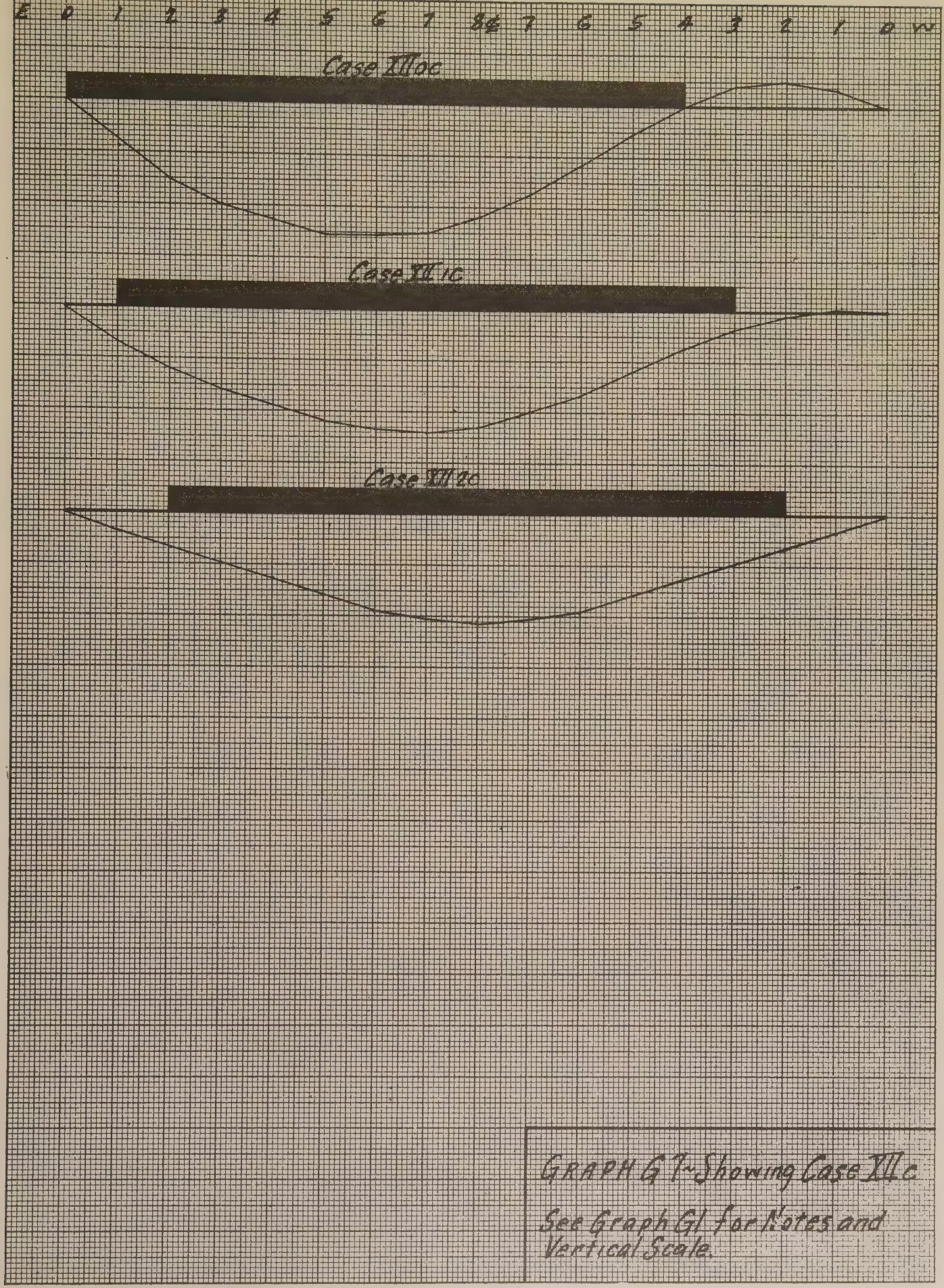
Graph G5: Showing Case VIII c

See Graph G1 for Notes and Vertical Scale.



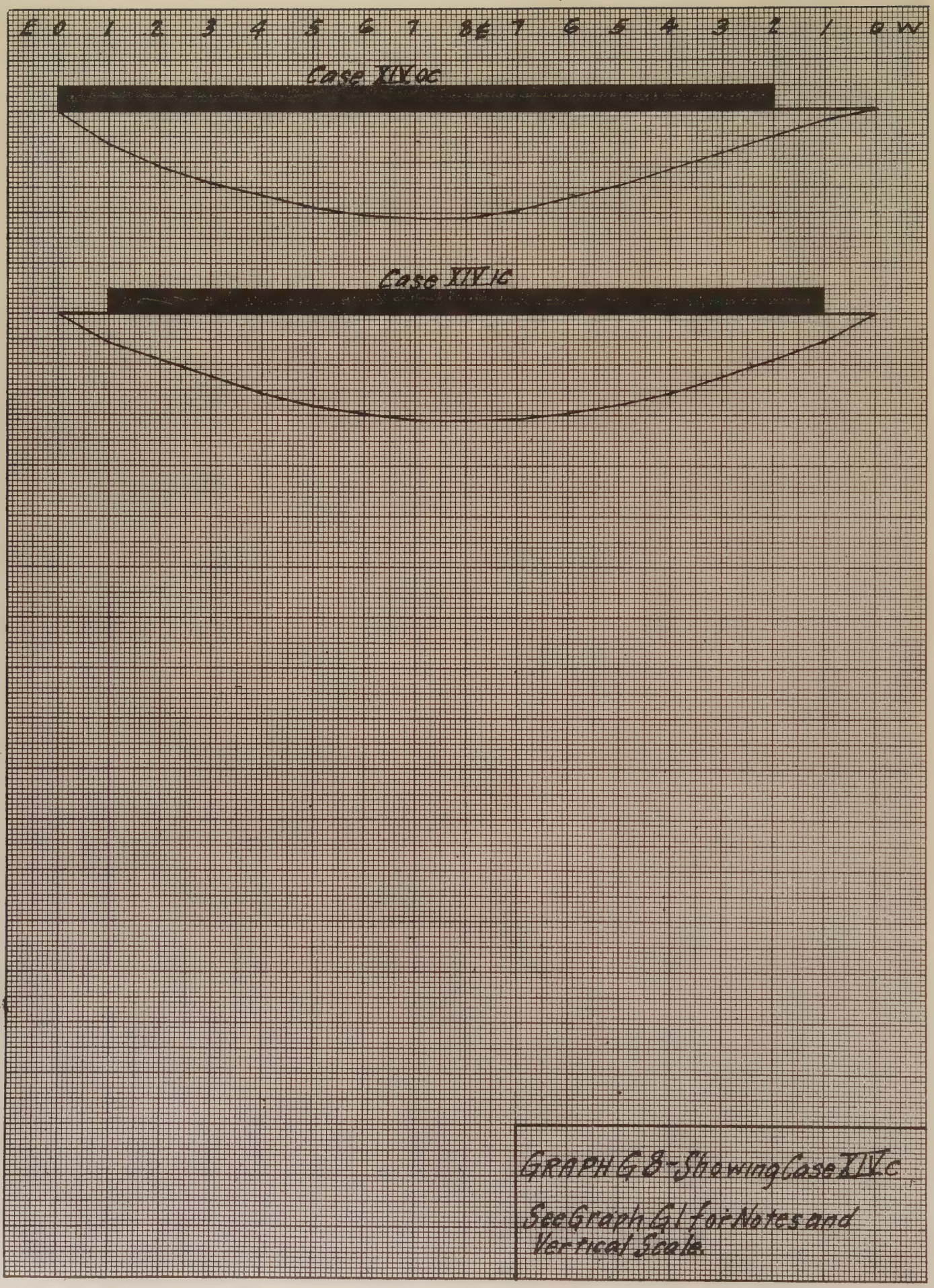
Graph G6 - Showing Case X c

See Graph G1 for Notes and Vertical Scale.



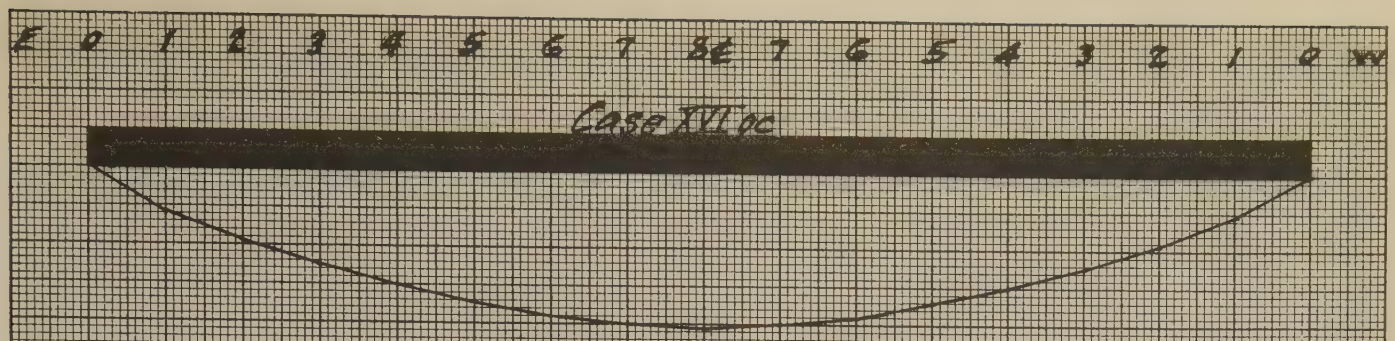
GRAPH G 7 - Showing Case XII c.

See Graph G1 for Notes and Vertical Scale.



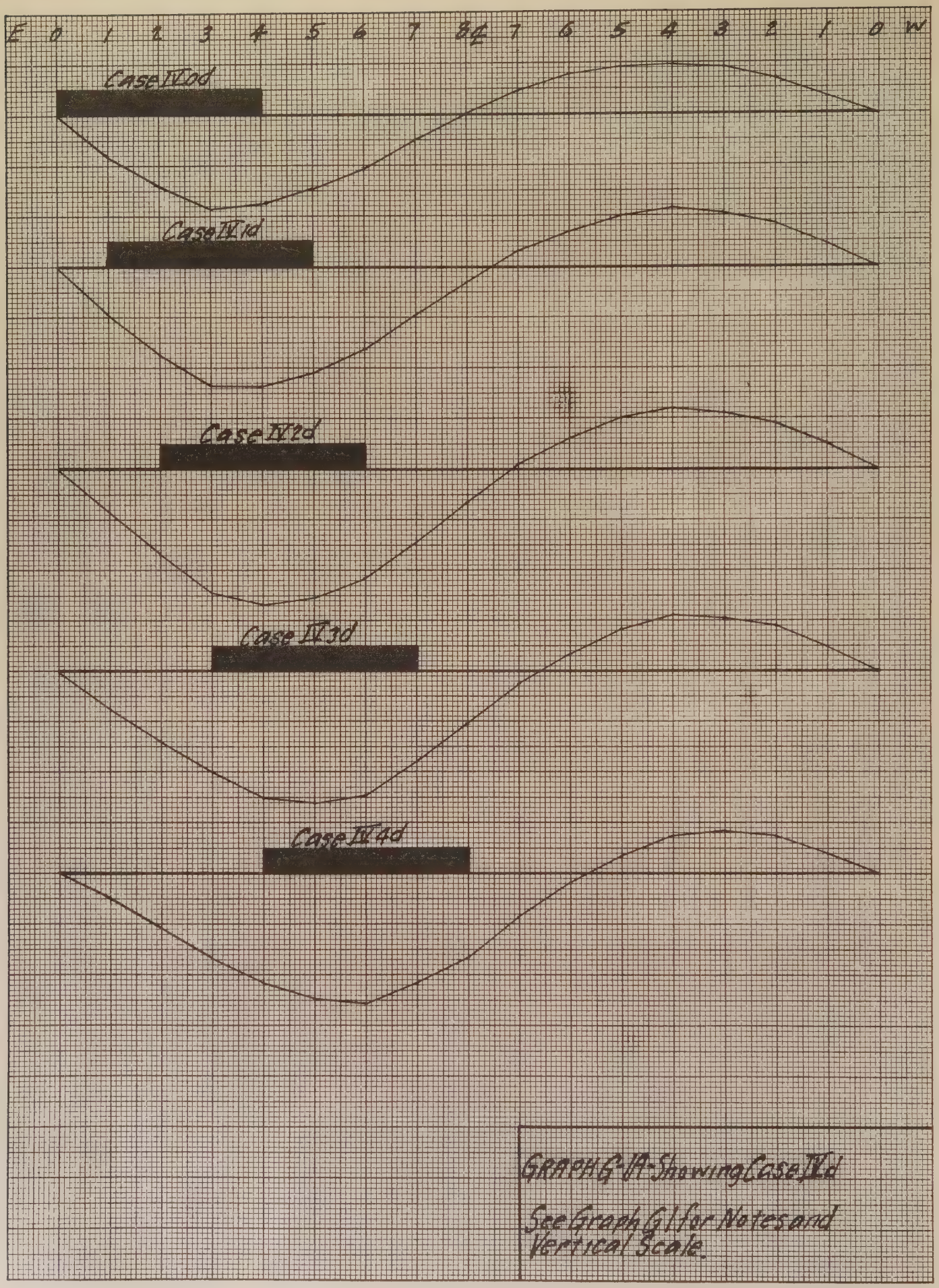
GRAPH G 8 - Showing Case XIV C

See Graph G 1 for Notes and Vertical Scale.



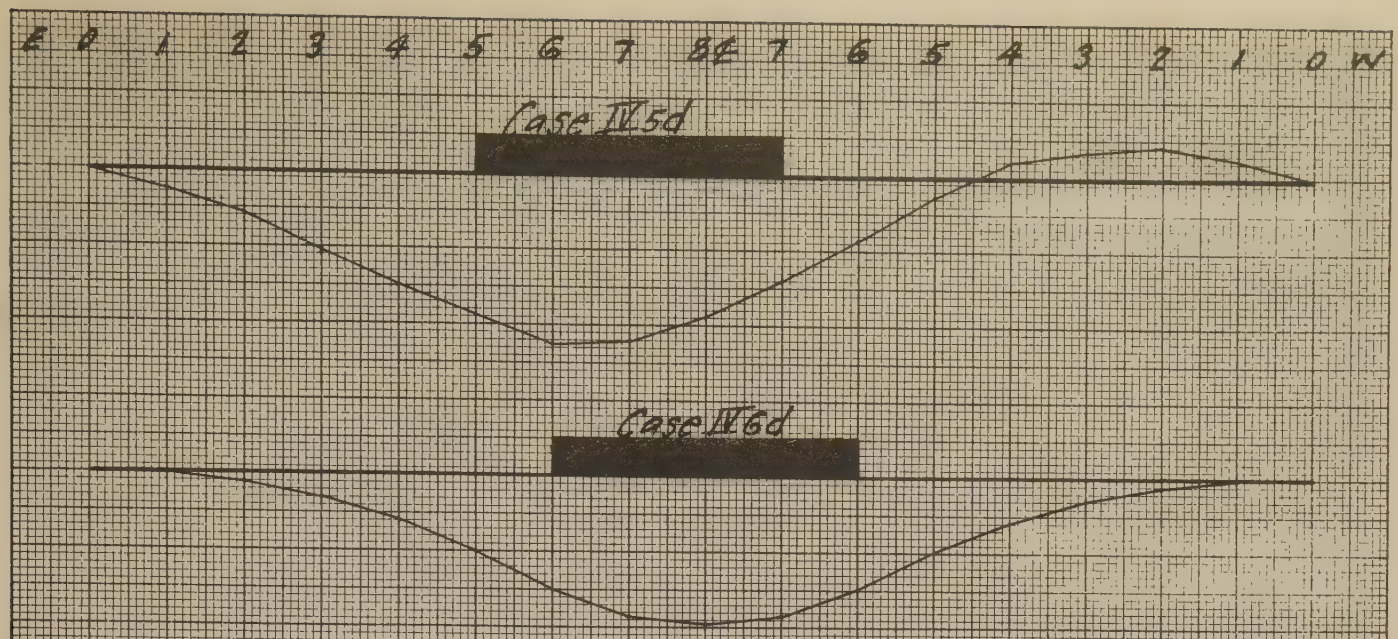
GRAPH G 9- Showing Case XVI c

See Graph G1 for Notes and Vertical Scale.



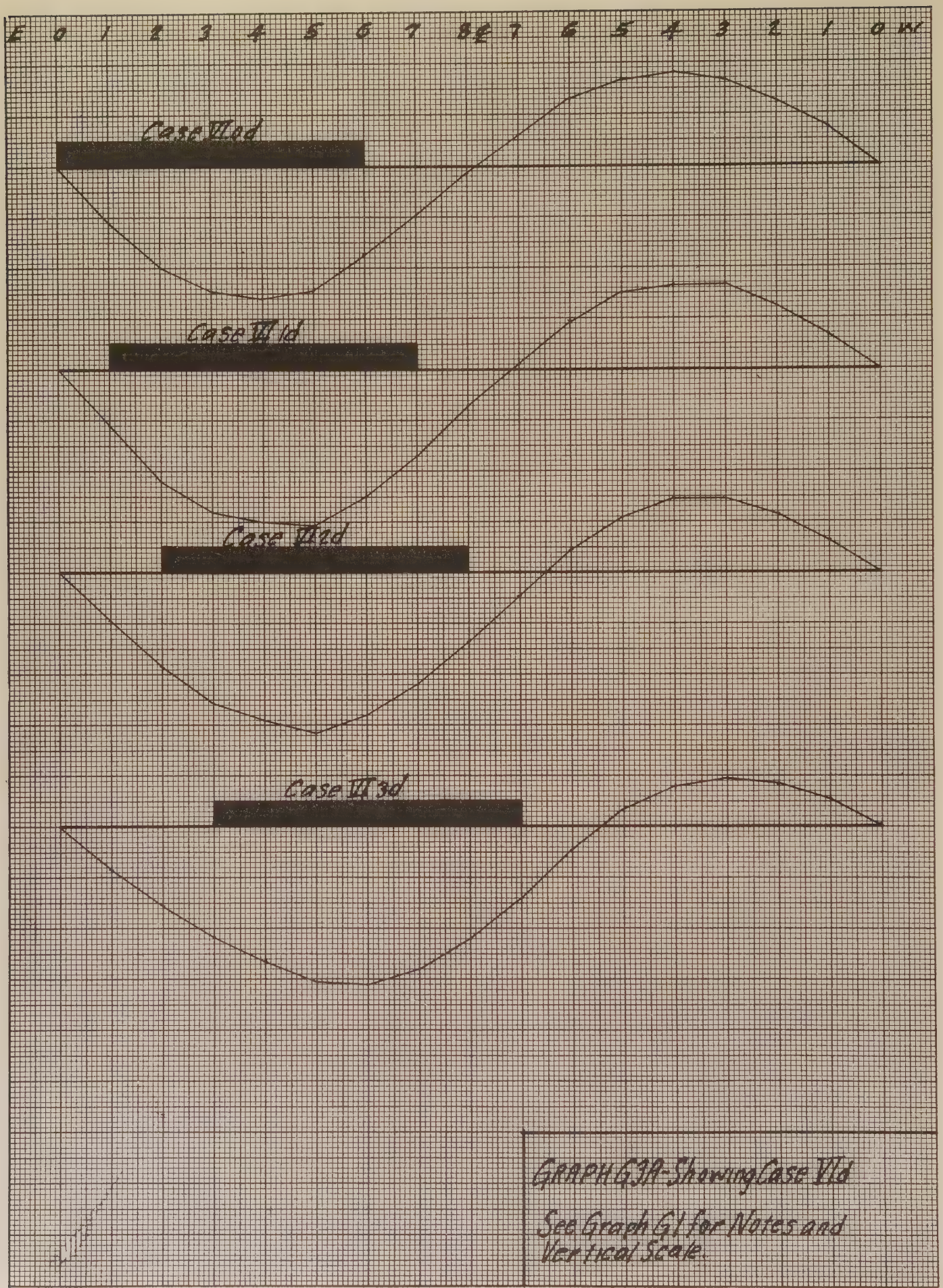
GRAPH G2 #1 - Showing Case IV d

See Graph G1 for Notes and Vertical Scale.



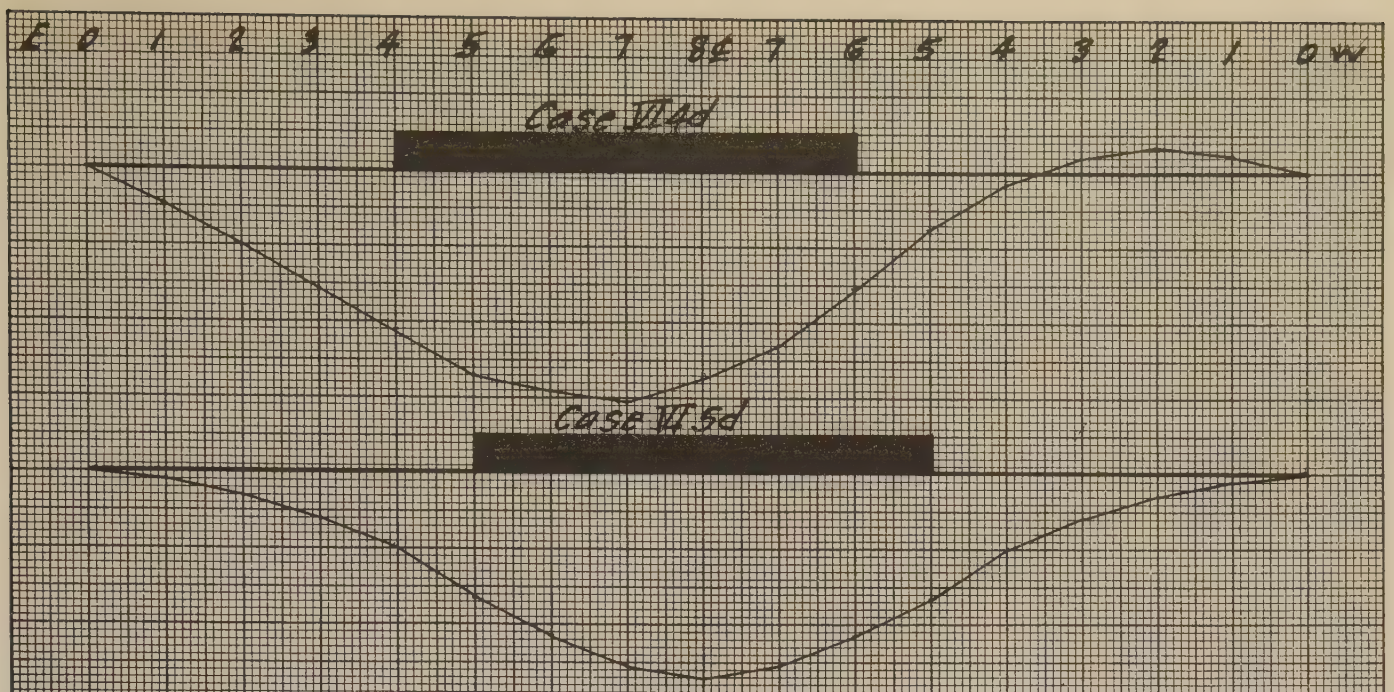
GRAPH 6 2A- Showing Case IV d Cont.

See Graph 6 1 for Notes and Vertical Scale.



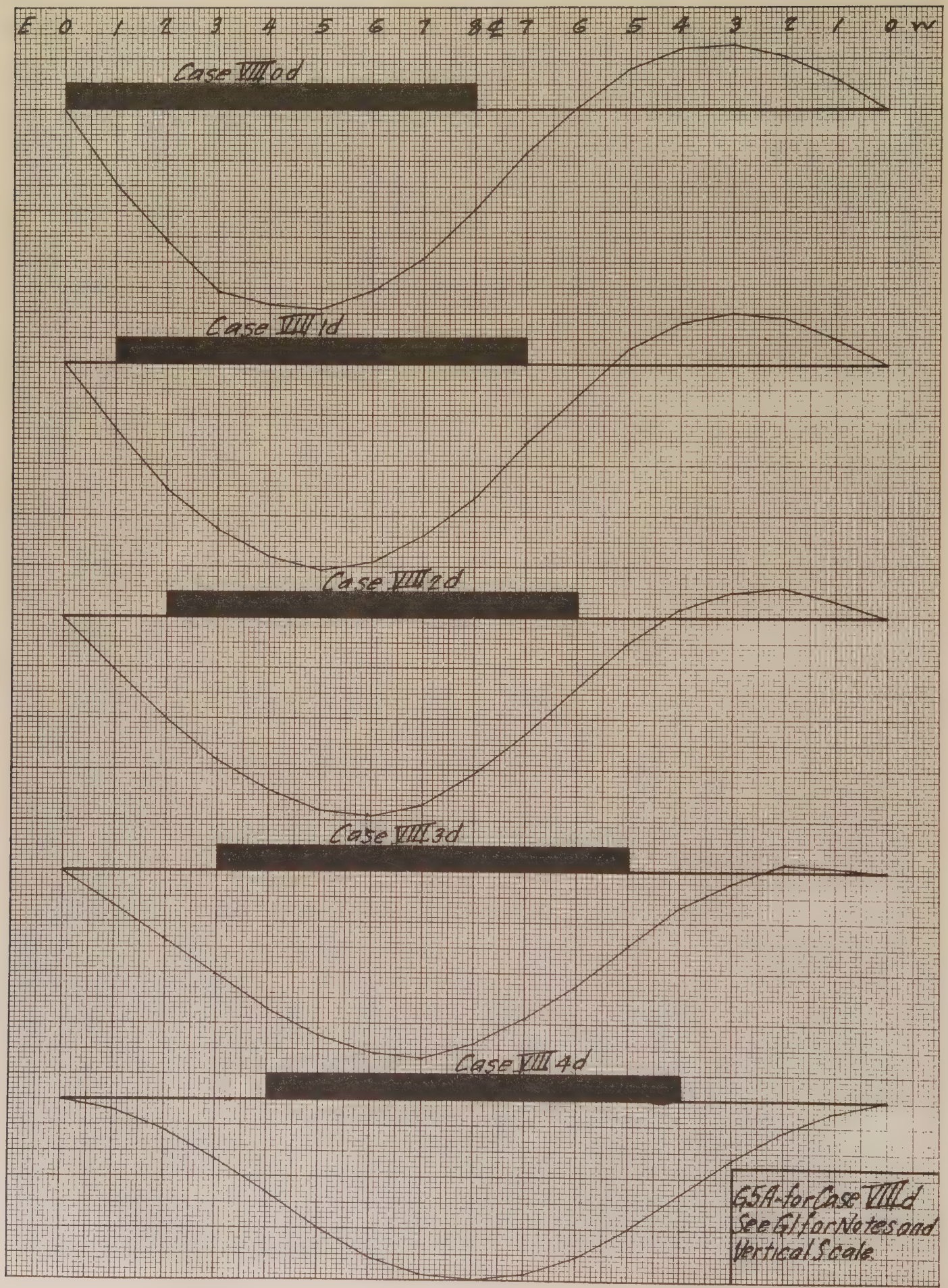
GRAPH G3R-Showing Case VIIIa

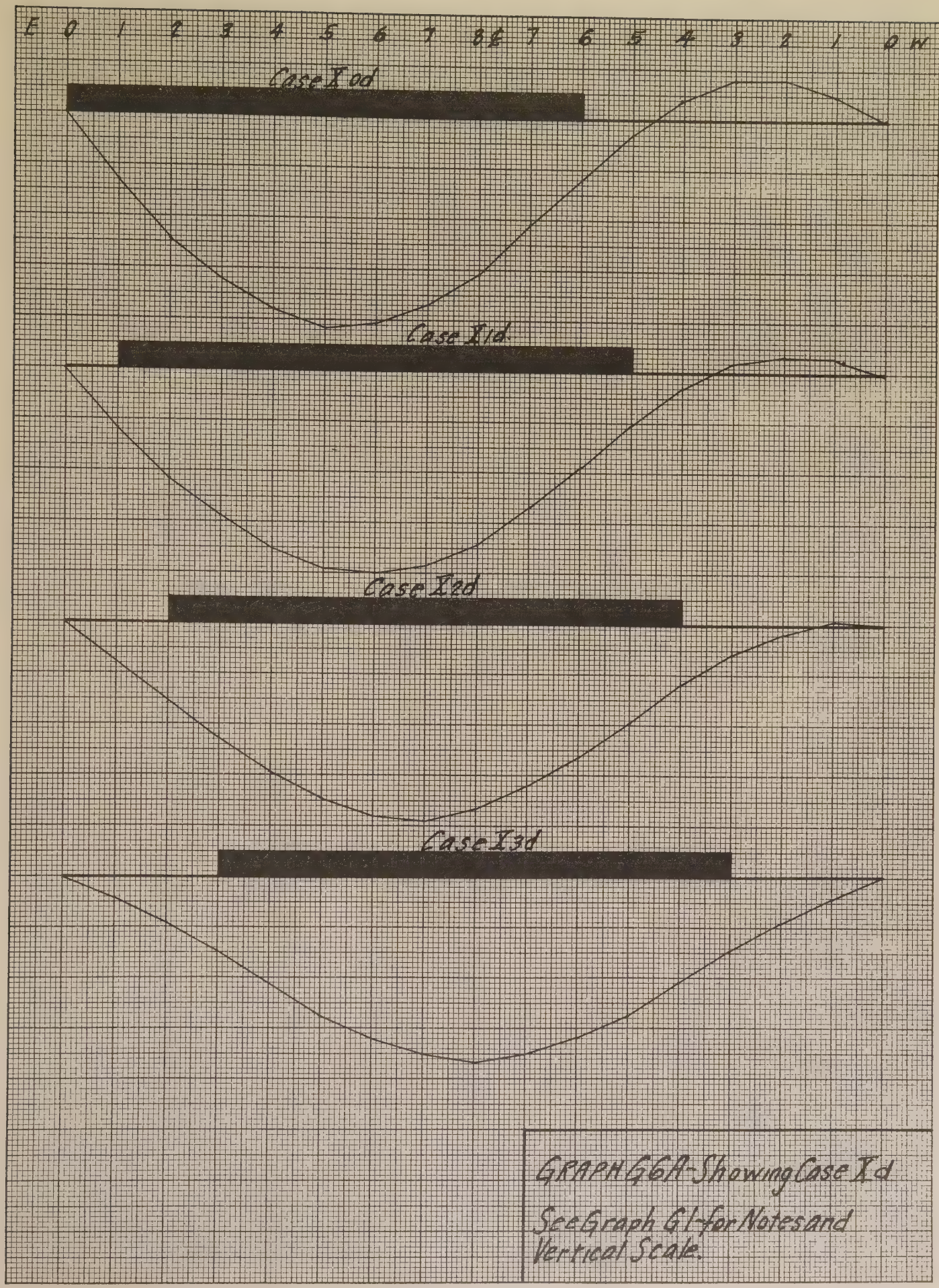
See Graph G1 for Notes and Vertical Scale.



GRAPH G 48 - Showing Case VIIdCont.

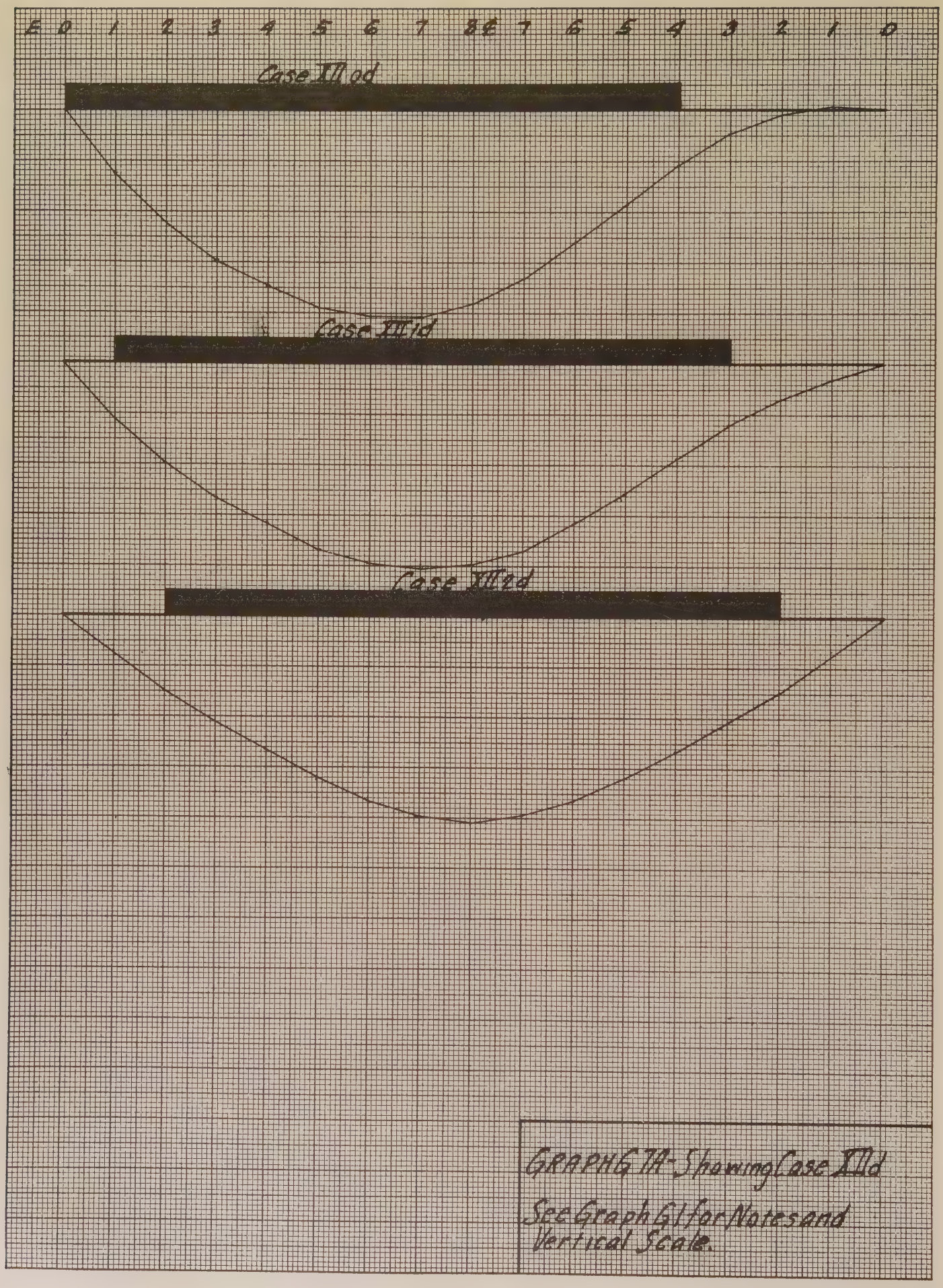
See Graph G1 for Notes and Vertical Scale


E 0 1 2 3 4 5 6 7 8 7 6 5 4 3 2 1 0 W
Case VIII 0d
Case VIII 1d
Case VIII 2d
Case VIII 3d
Case VIII 4d
G5A-for Case VIII d
See G1 for Notes and
Vertical Scale.



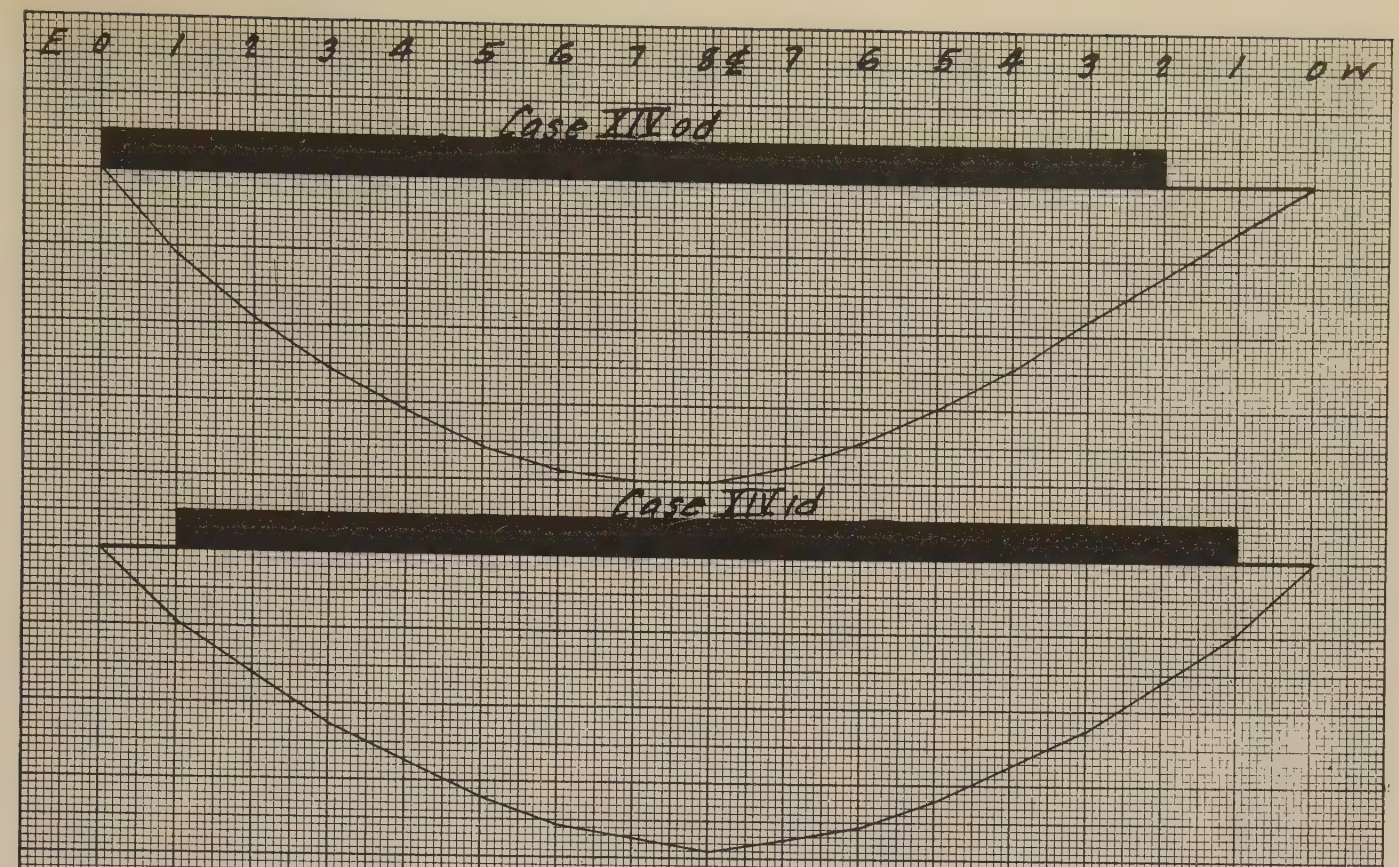
GRAPH G6A-Showing Case X d

See Graph G1 for Notes and Vertical Scale.



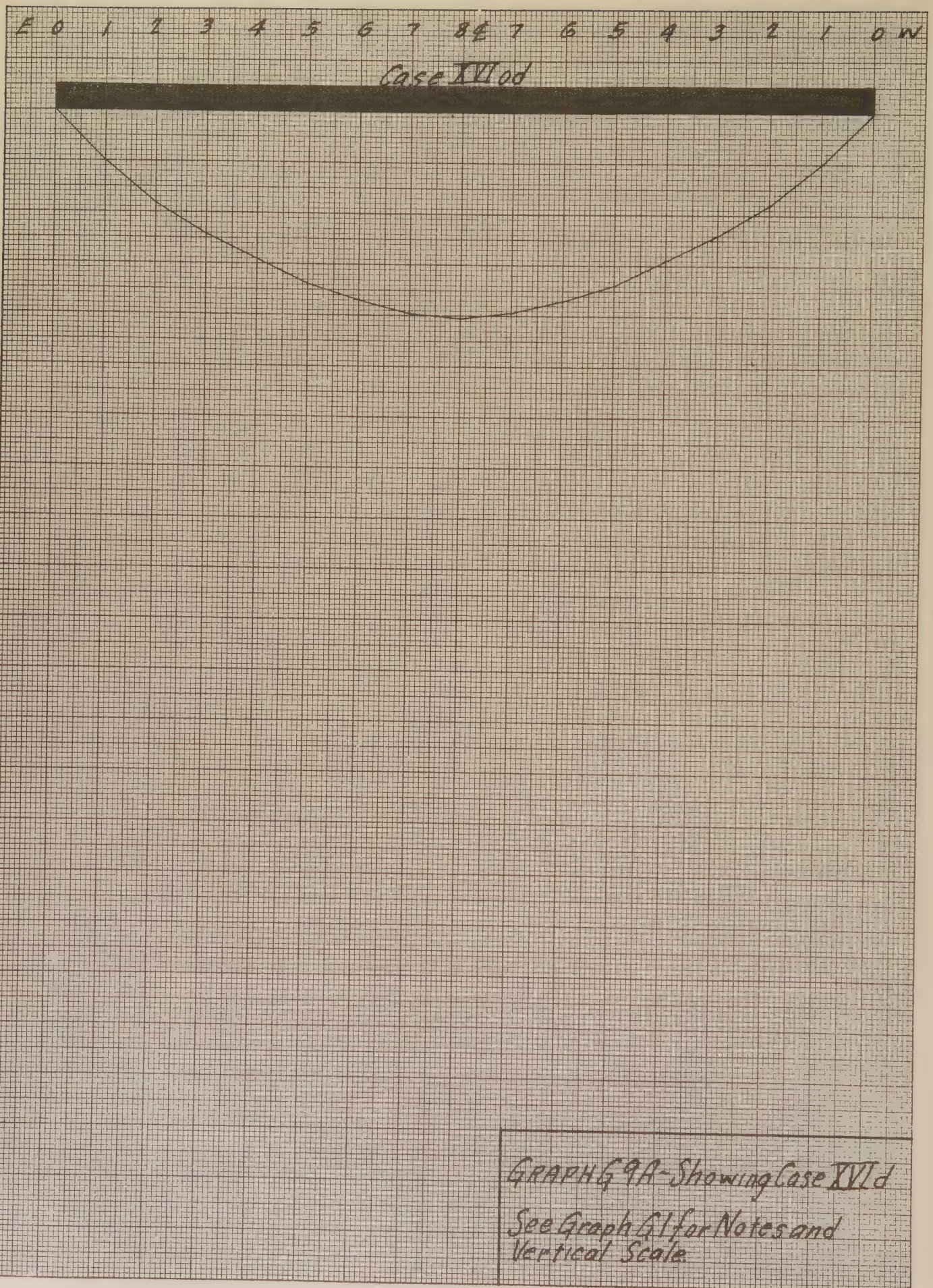
GRAPH G III - Showing Case IIId

See Graph G1 for Notes and Vertical Scale.



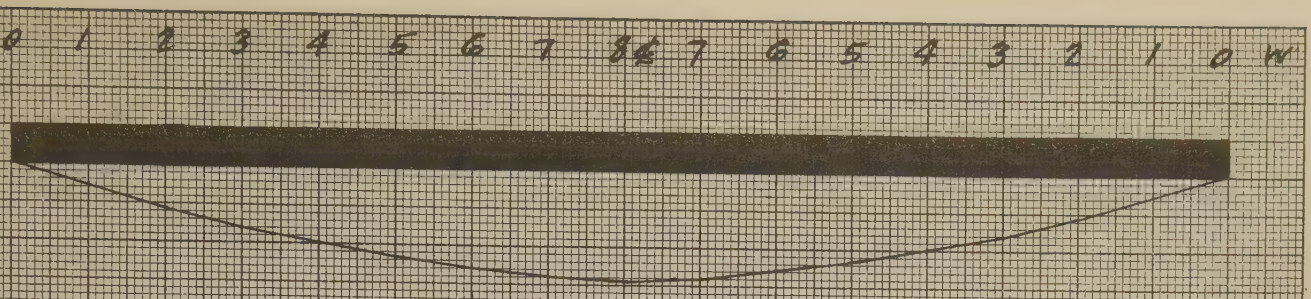
GRAPH G 8A - Showing Case XIV d

See Graph G1 for Notes and Vertical Scale.



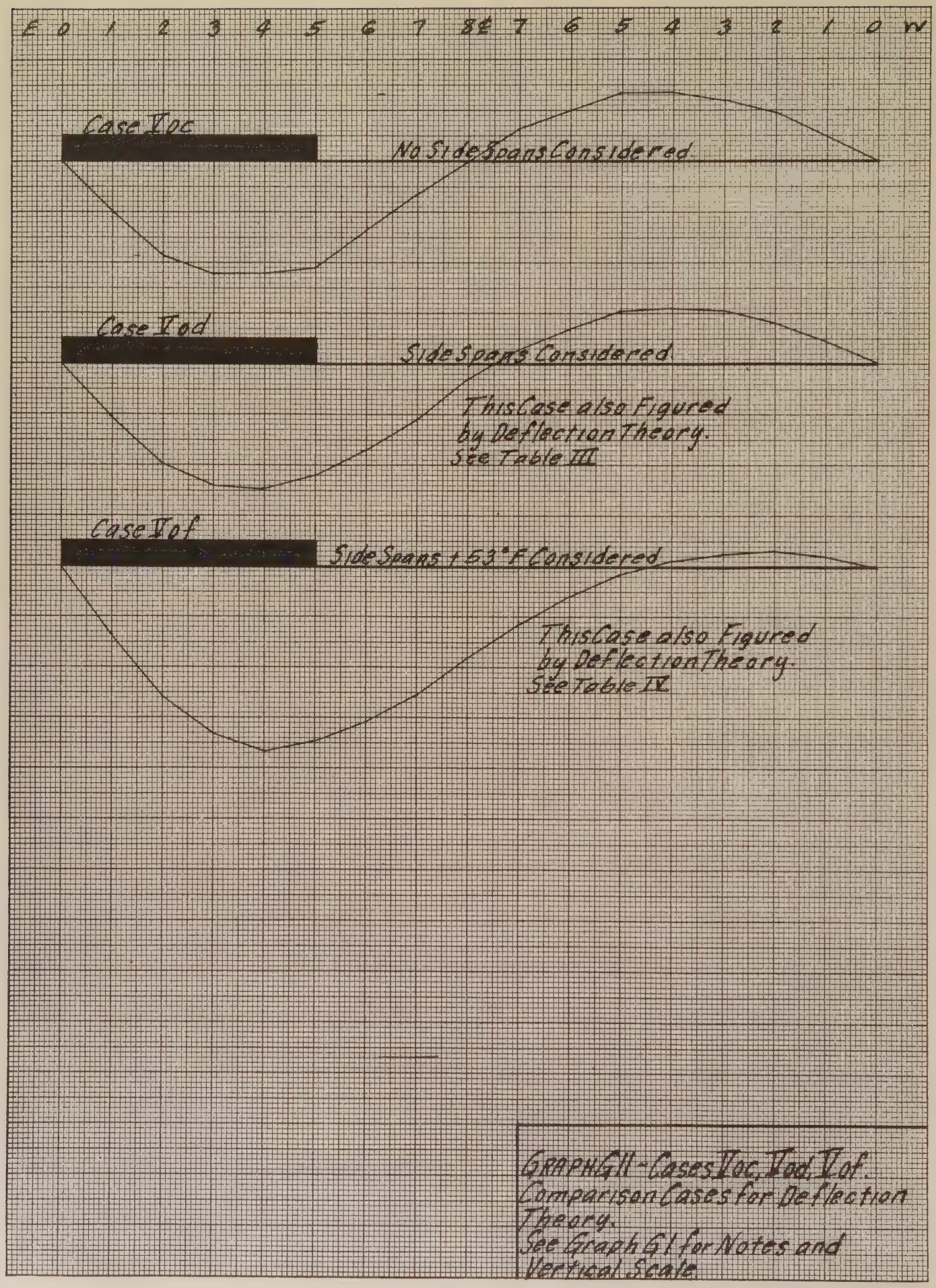
GRAPH G 9R - Showing Case XVI d

See Graph G 1 for Notes and Vertical Scale.

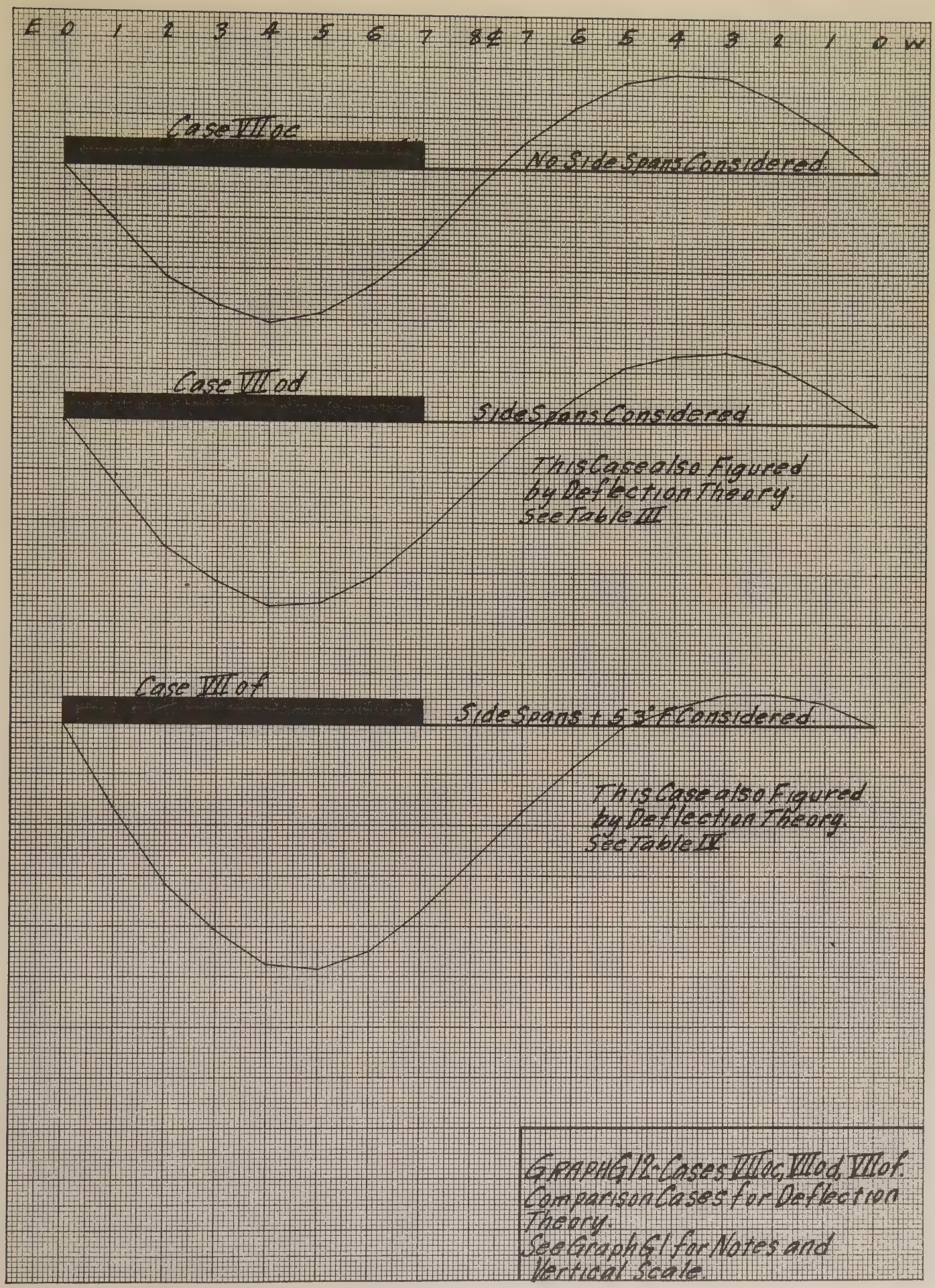


Temperature shown as a hypothetical load. This is the only case where weight had to be removed from the dead load counter-weight.

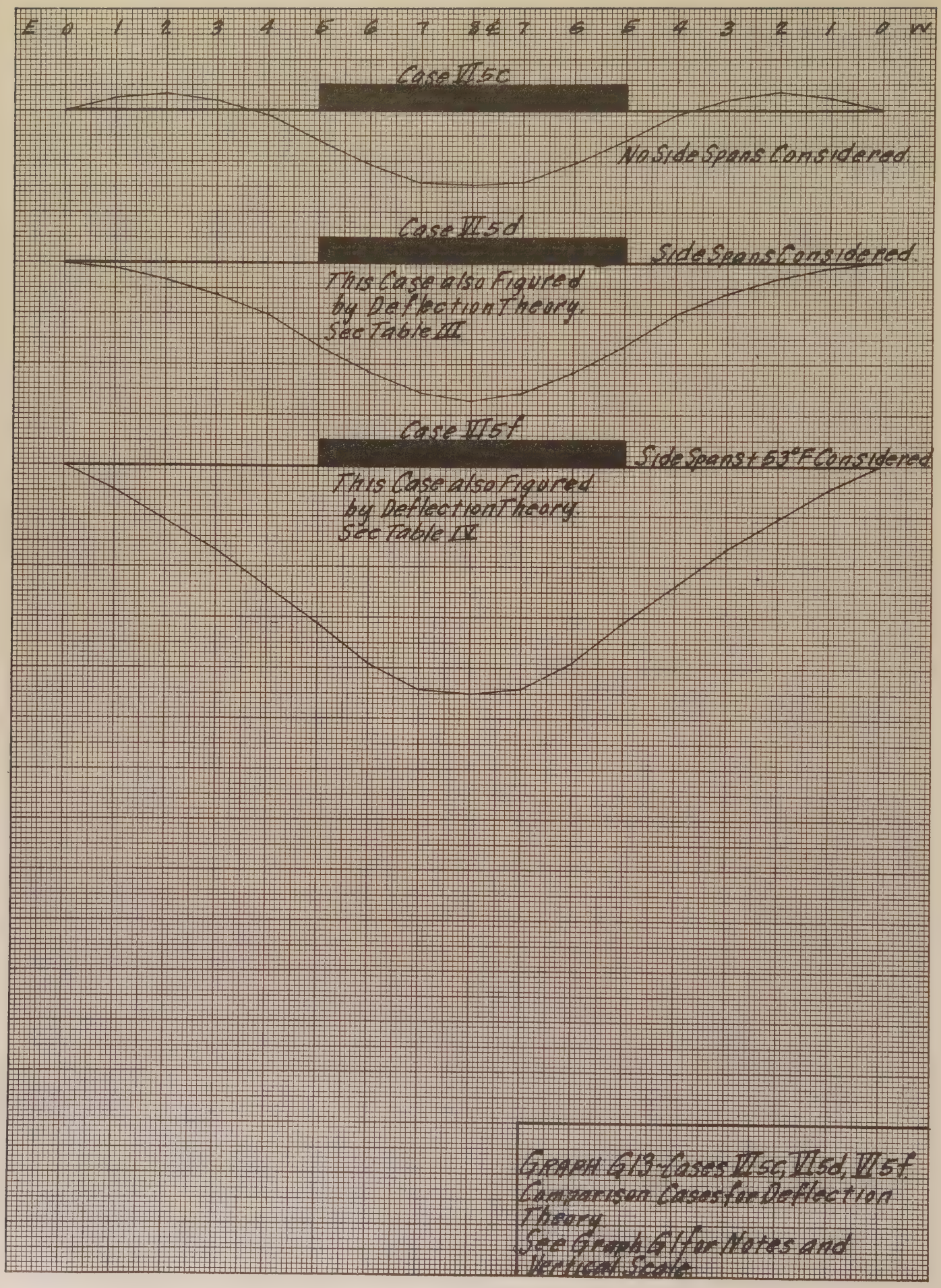
GRAPH G10 Temperature Alone of +53° F (3 SPANS CONSIDERED). See Graph G1 for Notes and Vertical Scale.

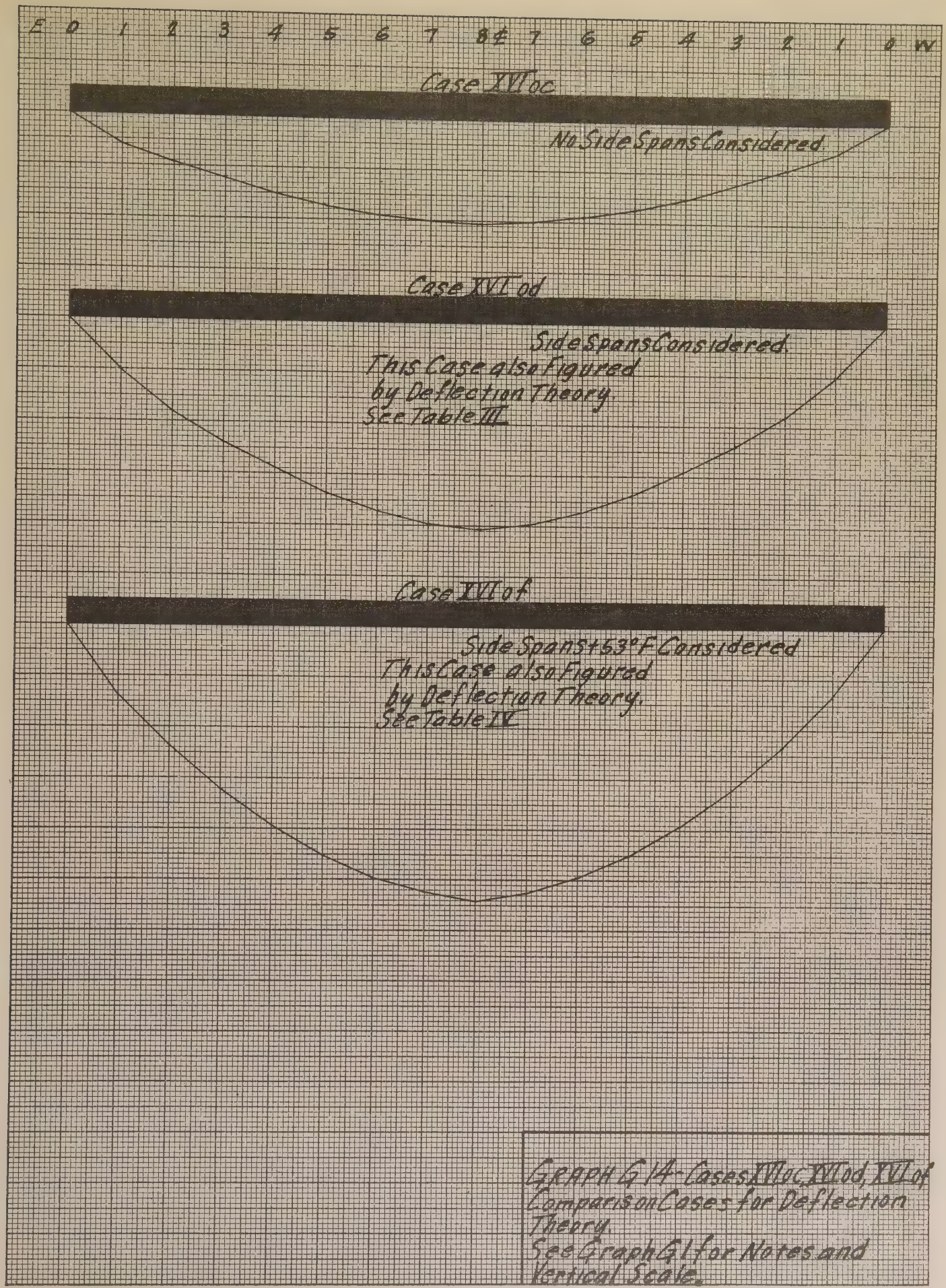

E 0 1 2 3 4 5 6 7 8E 7 6 5 4 3 2 1 0 W
Case V oc
No Side Spans Considered
Case V od
Side Spans Considered
This Case also Figured by Deflection Theory. See Table III
Case V of
Side Spans + 53° F Considered
This Case also Figured by Deflection Theory. See Table IV

GRAPH G11 - Cases V oc, V od, V of. Comparison Cases for Deflection Theory. See Graph G1 for Notes and Vertical Scale.

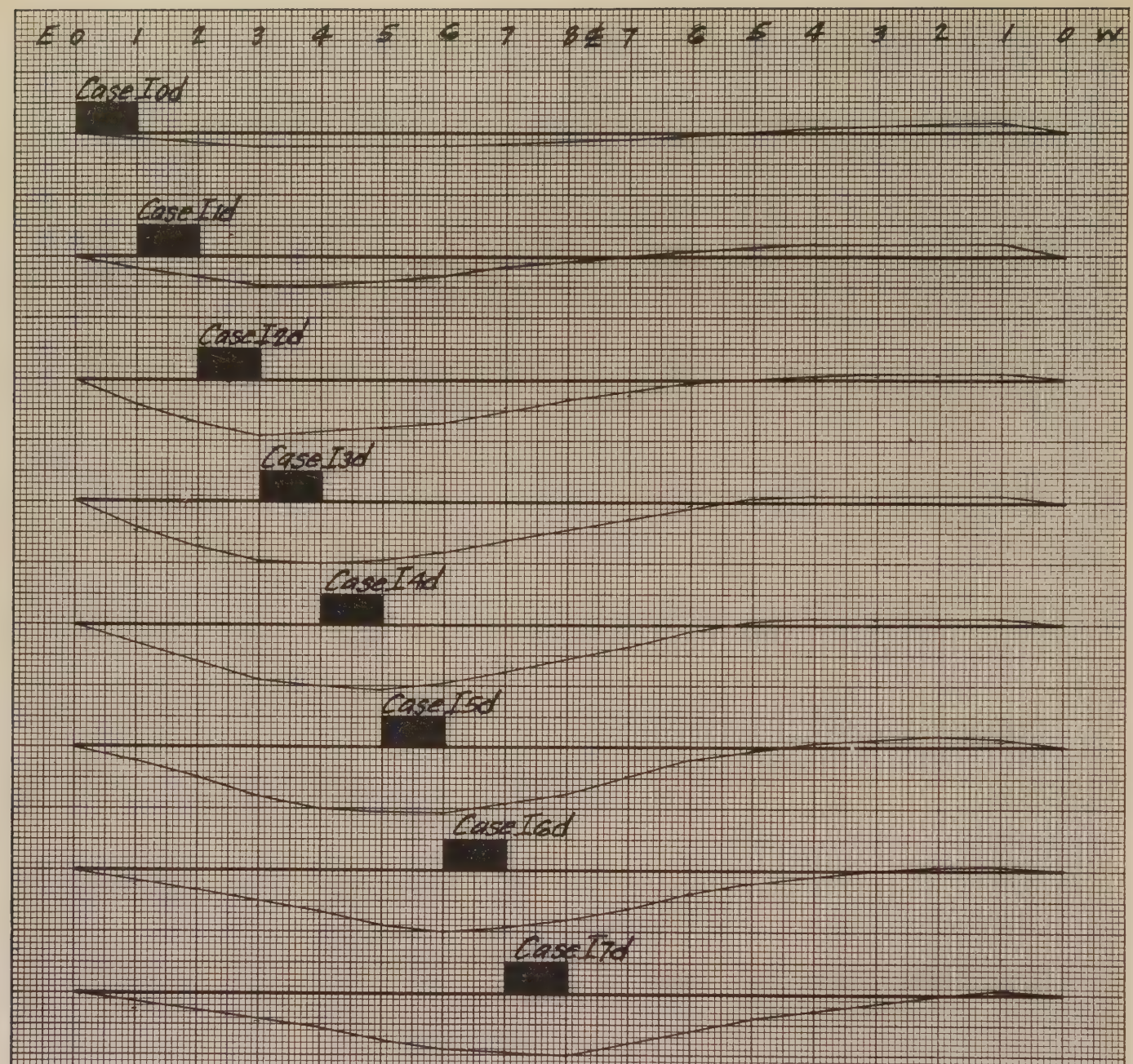

E 0 1 2 3 4 5 6 7 8 7 6 5 4 3 2 1 0 W
Case VIII oc
No Side Spans Considered.
Case VIII od
Side Spans Considered.
This Case also Figured by Deflection Theory. See Table III.
Case VIII of
Side Spans 1 & 3 F Considered.
This Case also Figured by Deflection Theory. See Table IV.

GRAPH G12. Cases VIIIoc, VIIIod, VIIIof. Comparison Cases for Deflection Theory. See Graph G1 for Notes and Vertical Scale.

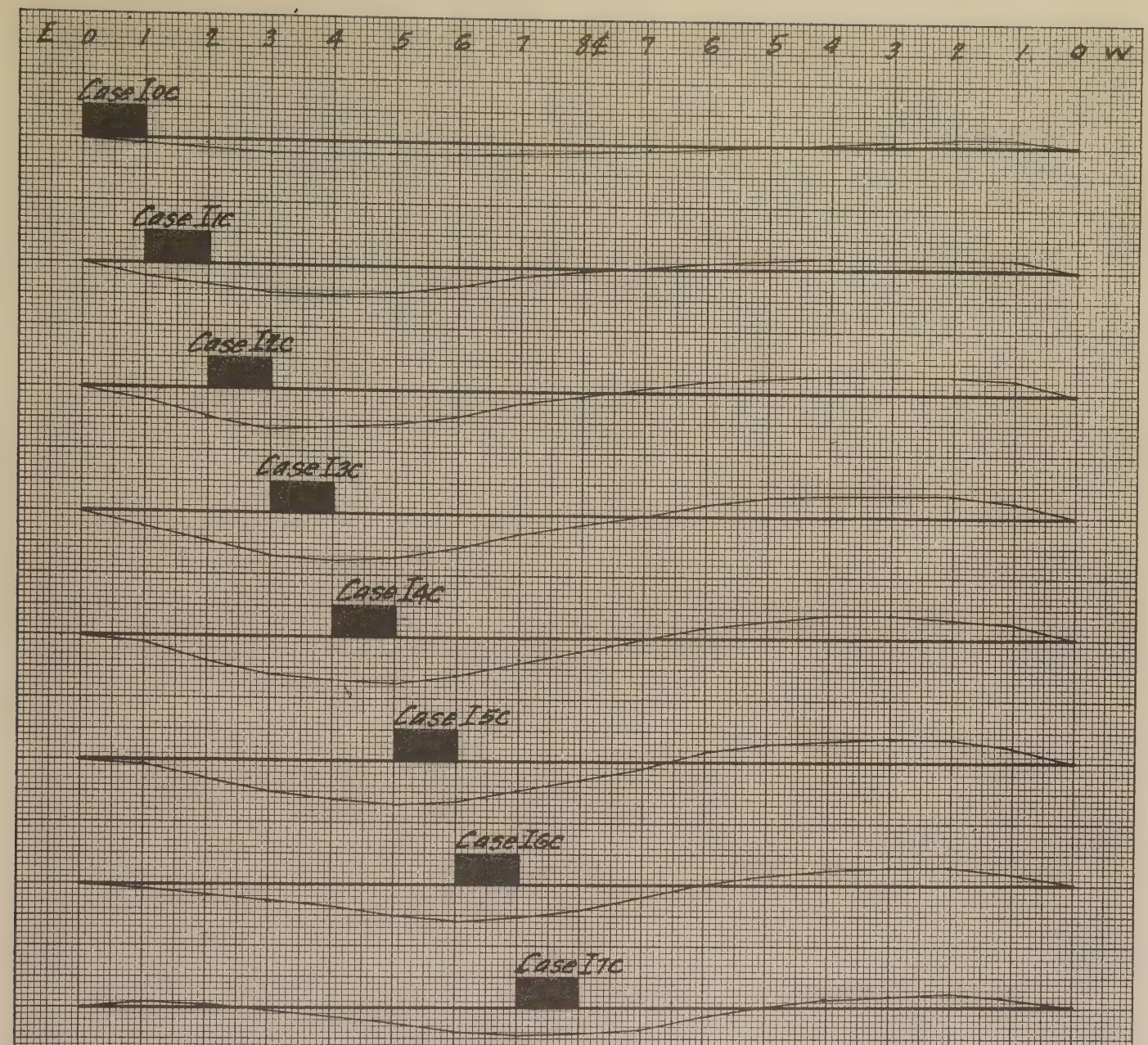

E 0 1 2 3 4 5 6 7 8 7 6 5 4 3 2 1 0 W
Case VI 5c
No Side Spans Considered.
Case VI 5d
Side Spans Considered.
This Case also Figured by Deflection Theory. See Table III
Case VI 5f
Side Spans t 63°F Considered
This Case also Figured by Deflection Theory. See Table IX
Graph 613 - Cases VI 5c, VI 5d, VI 5f. Comparison Cases for Deflection Theory. See Graph 611 for Notes and Vertical Scale.


E 0 1 1 2 3 4 5 6 7 8 E 7 6 5 4 3 2 1 0 W
Case XVI oc
No Side Spans Considered
Case XVI od
Side Spans Considered.
This Case also figured
by Deflection Theory.
See Table III
Case XVI of
Side Spans +63° F Considered
This Case also Figured
by Deflection Theory.
See Table IV

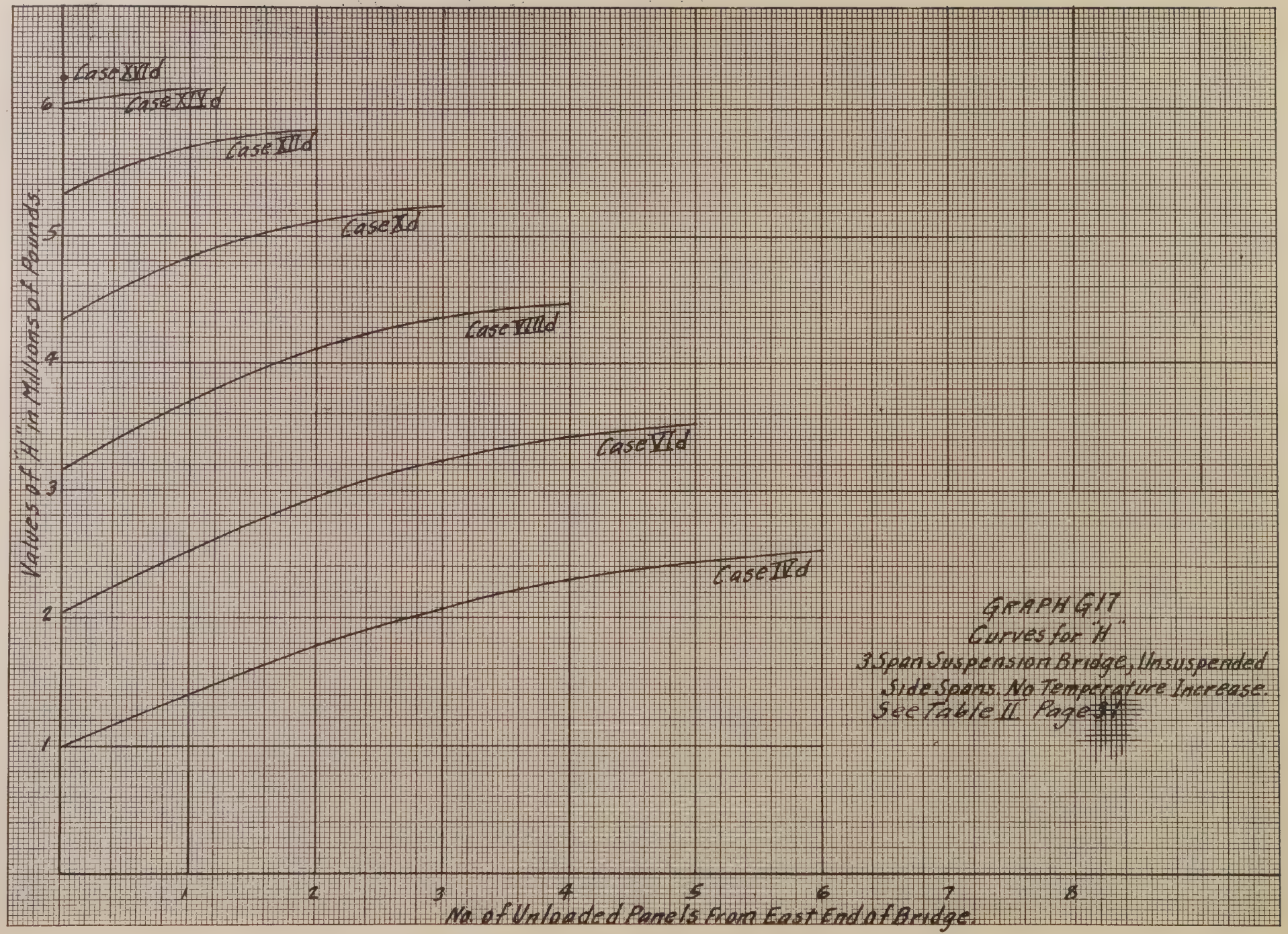
GRAPH G 14 - Cases XVI oc, XVI od, XVI of
Comparison Cases for Deflection
Theory.
See Graph G1 for Notes and
Vertical Scale.



GRAPH G15 - Showing Case Id.
For Qualitative Uses Only.
See Graph G1 for Notes and Vertical Scale



GRAPH G16 - Showing Case Ic.
For Qualitative Uses Only.
See Graph G1 for Notes and
Vertical Scale.


Case XIIId
Case XIVd
Case XIId
Case Xd
Case VIIId
Case VId
Case IVd
Values of "H" in Millions of Pounds
1
2
3
4
5
6
1
2
3
4
5
6
7
8
No. of Unloaded Panels from East End of Bridge.
GRAPH G17
Curves for "H"
3 Span Suspension Bridge, Unsuspended
Side Spans. No Temperature Increase.
See Table II. Page 34

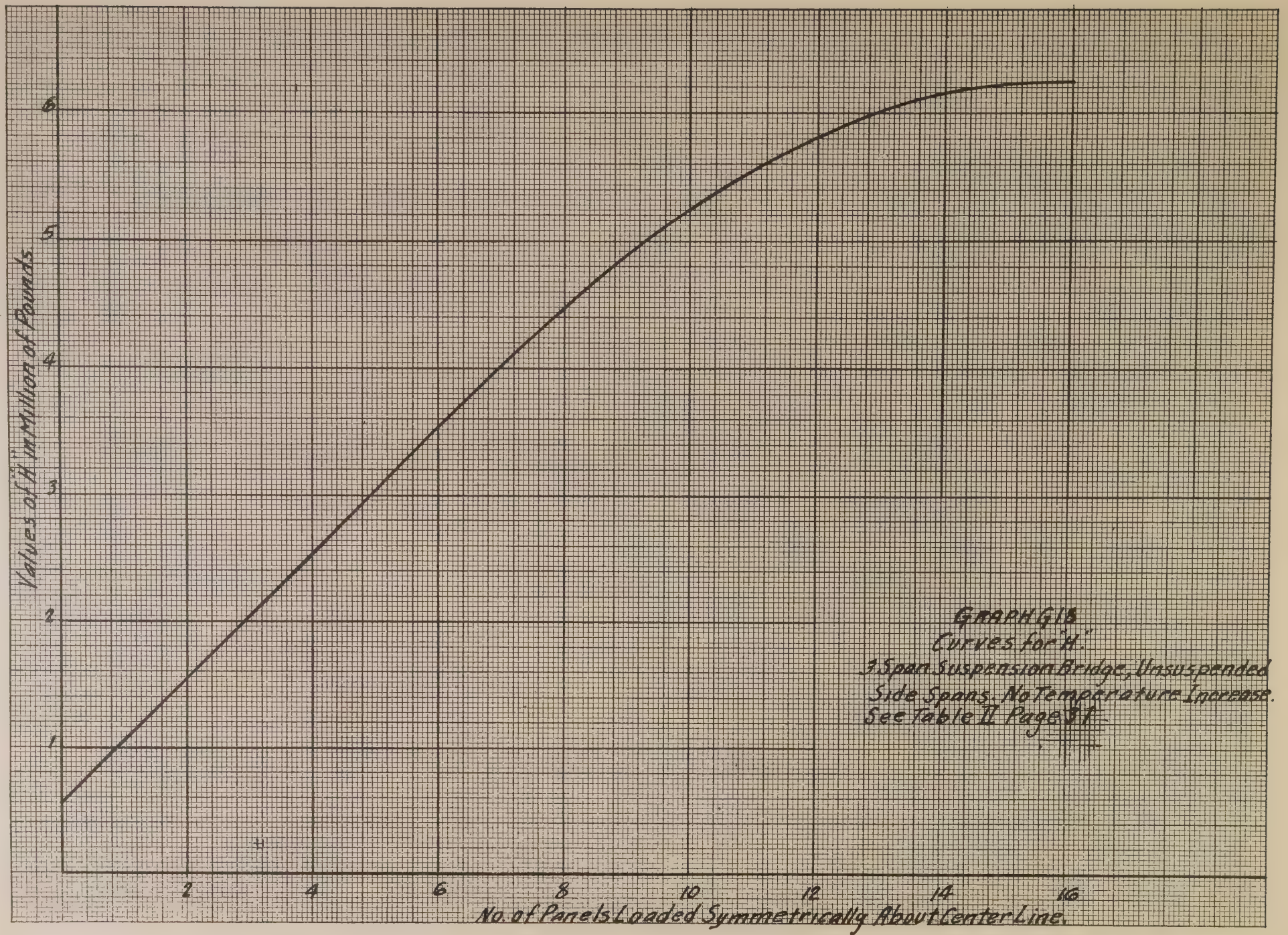
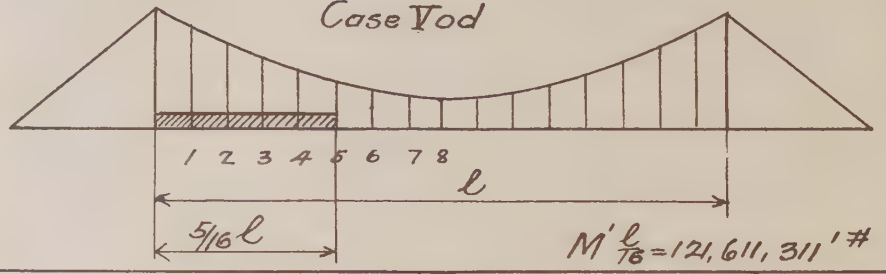
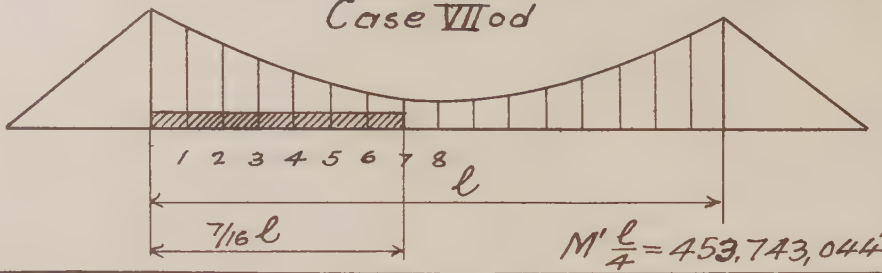
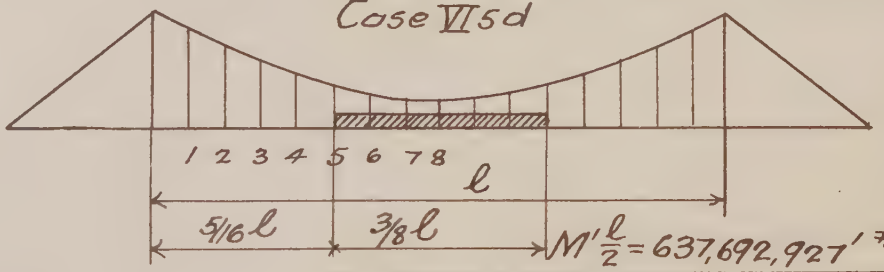
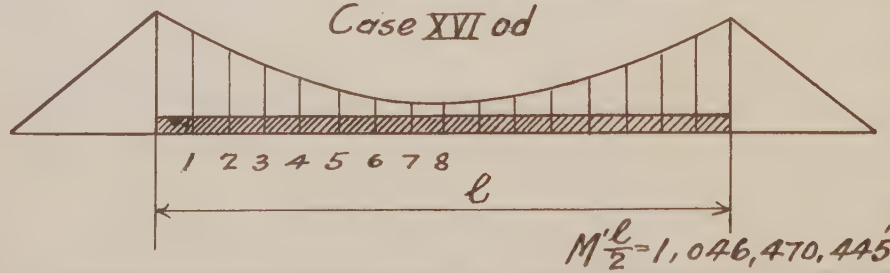
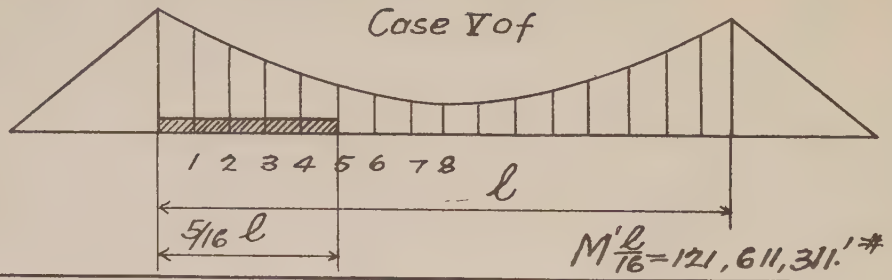
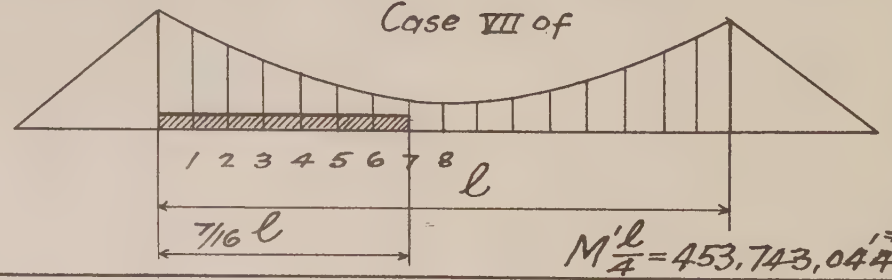
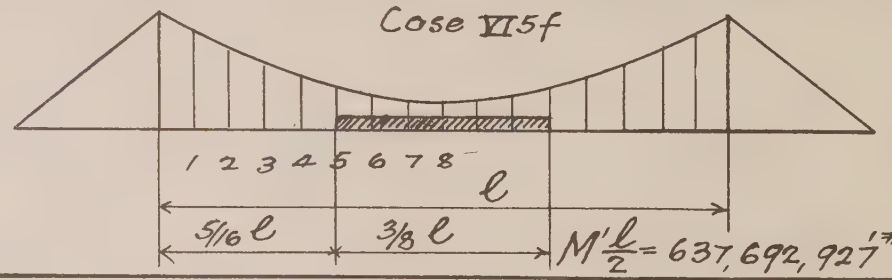
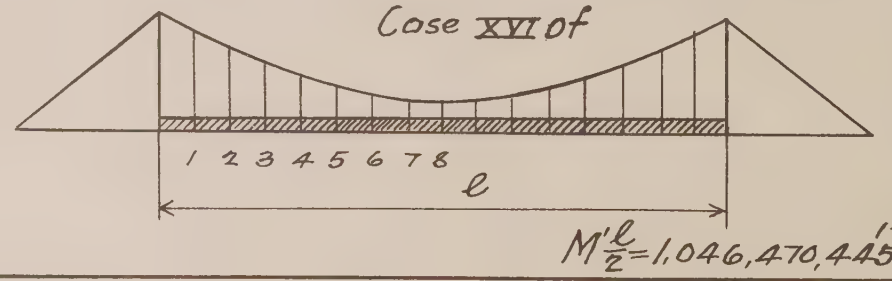

Values of "H" in Millions of Pounds
1
2
3
4
5
6
2
4
6
8
10
12
14
16
No. of Panels Loaded Symmetrically About Center Line.
GRAPH G15
Curves for "H".
3 Span Suspension Bridge, Unsuspended Side Spans. No Temperature Increase.
See Table II Page 54

TABLE III

Temperature Normal	Values by Deflection Theory	Values from Model
Case V od  $M'\frac{\ell}{16}$ = 121,611,311'#	H+Hw = 11,951,721#	H+Hw = 12,010,300#
	H = 1,471,721#	H = 1,441,300#
	Hw = 10,480,000#	Hw = 10,569,000#
	Deflection(n) p.pt 1 = 1.920'	Deflection(n) p.pt 1 = 1,680#
	Shear Left End = 717,781#	Shear Left End ● = 730,658#
	y = 34.055'	y = 35.147'
Case VII od  $M'\frac{\ell}{4}$ = 453,743,044'#	H+Hw = 13,107,812#	H+Hw = 13,133,900#
	H = 2,627,812#	H = 2,564,900#
	Hw = 10,480,000#	Hw = 10,569,000#
	Deflection(n) p.pt 4 = 5.161'	Deflection(n) p.pt 4 = 5.840
	Moment p.pt 4 = 99,663,698'#	Moment p.pt 4 ●= 96,616,681'#
	y = 109.000'	y = 109.328'
Case VI 5d  $M'\frac{\ell}{2}$ = 637,692,927'#	H+Hw = 14,041,100#	H+Hw = 14,118,380#
	H = 3,561,100#	H = 3,549,380#
	Hw = 10,480,000#	Hw = 10,569,000#
	Deflection(n) p.pt 8 = 4.139'	Deflection(n) p.pt 8 4.320'
	Moment p.pt 8 = 62,149,088'#	Moment p.pt 8 ●= 60,976,611'#
	y = 145.300'	y = 145.300'
Case XVI od  $M'\frac{\ell}{2}$ = 1,046,470,445'#	H+Hw = 16,871,725#	H+Hw = 16,817,593#
	H = 6,391,725#	H = 6,248,593#
	Hw = 10,480,000#	Hw = 10,569,000#
	Deflection(n) p.pt 8 = 5.254'	Deflection(n) p.pt 8 = 6.480'
	Moment p.pt 8 = 29,108,760'#	Moment p.pt 8 ●= 29,571,879'#
	y = 145.300'	y = 145.300'

● Shear deduced from Moment at p.pt 1. ●— From Fundamental Formula $M = M' - (H+Hw)n - H(y)$, using Data from Model. Differences of "y" values given in "Deflection Theory" column and in "from Model" column are due to slight variations in panel spacing of Model from theoretical figures as explained on page 8 and are negligible.

TABLE IV

Temperature Normal + 53° F	Values by Deflection Theory	Values from Model
	H+Hw = 11,706,600#	H+Hw = 11,717,900#
	H = 1,226,600#	H = 1,148,900#
	Hw = 10,480 000#	Hw = 10,569,000#
	Deflection(n) p.pt 1 = 2.314'	Deflection(n) p.pt 1 = 2.160'
	Shear Left End = 764,265#	Shear Left End = 810,604#
	y = 34.055'	y = 35.147'
	H+Hw = 12,858,144#	H+Hw = 12,811,900#
	H = 2,378,144#	H = 2,242,900#
	Hw = 10,480,000#	Hw = 10,569,000#
	Deflection(n) p.pt 4 = 6 613'	Deflection(n) p.pt 4 = 7.606'
	Moment p.pt 4 = 109,489,700'#	Moment p.pt 4 = 111,160,833'#
	y = 109.000'	y = 109.328'
	H+Hw = 13,787,948#	H+Hw = 13,865,200#
	H = 3,307,948#	H = 3,296,200#
	Hw = 10,480,000#	Hw = 10,569,000#
	Deflection(n) p.pt 8 = 6,083'	Deflection(n) p.pt 8 = 6.200'
	Moment p.pt 8 = 73,182,325'#	Moment p.pt 8 = 72,790,827'#
	y = 145.300'	y = 145.300'
	H+Hw = 16,581,586#	H+Hw = 16,523,862#
	H = 6,101.586#	H = 5,954,862#
	Hw = 10,480,000#	Hw = 10,569,000#
	Deflection(n) p.pt 8 = 7.281'	Deflection(n) p.pt 8 = 8.640'
	Moment p.pt 8 = 39,179,471'#	Moment p.pt 8 = 38,462,828'#
	y = 145.300'	y = 145.300'

See under TABLE III Page 60 for footnotes

Case V od Shear at Left End of Span See Graph G11.

H+Hw = 12,010,300#
H = 1,441,300#
n = 1.680'
$M'\frac{\ell}{16}$ = 121,611,311#
y = 35.1472'

(H+Hw)n = 20,177,304
H(y) = 50,657,659
70,834,963

121,611,311
70,834,963
50,776,348

$\frac{4000x(93.344)^2}{2}$ = 17,426,205
68,202,553
93.344 = 730,658#

Case VII od Moment at $\frac{\ell}{4}$ See Graph G12.

H+Hw = 13,133,900#
H = 2,564,900#
n = 5.840'
$M'\frac{\ell}{4}$ = 453,734,044'#
y = 109.328'

(H+Hw)n = 76,701,976
H(y) = 280,415,387
357,117,363

453,734,044
357,117,363
96,616,681'#

Case VI 5d Moment at $\frac{\ell}{2}$ See Graph G13.

H+Hw = 14,118,380#
H = 3,549,380#
n = 4.32'
$M'\frac{\ell}{2}$ = 637,692,927'#
y = 145.3'

(H+Hw)n = 60,991,402
H(y) = 515,724,914
576,716,316

637,692,927
576,716,316
60,976,611'#

Case XVI od Moment at $\frac{\ell}{2}$ See Graph G14.

H+Hw = 16,817,593#
H = 6,248,593#
n = 6.48'
$M'\frac{\ell}{2}$ = 1,046,470,445#
y = 145.3'

(H+Hw)n = 108,978,003
H(y) = 907,920,563
1,016,898,566

1,046,470,445
1,016,898,566
29,571,879 #

Case V of Shear at Left End of Span See Graph G11.

H+Hw = 11,717,900#
H = 1,148,900#
n = 2.16'
$M'\frac{\ell}{16}$ = 121,611,311'#
y = 35.1472'

(H+Hw)n = 25,310,664
H(y) = 38,061,808
63,372,472

121,611,311
63,372,472
58,238,839
17,426,205
75,665,044

$\frac{4000x(93.344)^2}{2}$ = $\frac{75,665,044}{93.344}$ = 810,604#

Case VII of Moment at $\frac{\ell}{4}$ See Graph G12.

H+Hw = 12,811,900#
H = 2,242,900#
n = 7.60'
$M'\frac{\ell}{4}$ = 453,743,044'#
y = 109.328'

(H+Hw)n = 97,370,440
H(y) = 245,211,771
342,582,211

453,743,044
342,582,211
111,160,833'#

Case VI 5f Moment at $\frac{\ell}{2}$ See Graph G13.

H+Hw = 13,865,200#
H = 3,296,200#
n = 6.20'
$M'\frac{\ell}{2}$ = 637,692,927'#
y = 145.3'

(H+Hw)n = 85,964,240
H(y) = 478,937,860
564,902,100

637,692,927
564,902,100
72,790,827'#

Case XVI of Moment at $\frac{\ell}{2}$ See Graph G14.

H+Hw = 16,523,862#
H = 5,954,862#
n = 8.64'
$M'\frac{\ell}{2}$ = 1,046,470,445'#
y = 145.3'

(H+Hw)n 142,766,168
H(y) 865,241,449
1,008,007,617

1,046,470,445
1,008,007,617
38,462,828'#

CONCLUSIONS.

Tables III and IV give a complete summary of the eight cases selected for comparison, analysed by both model data and the conventional deflection theory.

For moments and shears, determined by the use of the model, variations from those found by the conventional deflection theory, expressed in percentages, are

Case V od	Shear at left end	1.8%
Case VII od	Moment at $\frac{l}{4}$	-3.1%
Case VI 5d	Moment at $\frac{l}{2}$	-1.9%
Case XVI od	Moment at $\frac{l}{2}$	1.6%
Case V of	Shear at left end	6.1%
Case VII of	Moment at $\frac{l}{4}$	1.5%
Case VI 5f	Moment at $\frac{l}{2}$	-0.54%
Case XVI of	Moment at $\frac{l}{2}$	-1.8%

It will be seen from the above percentages that the values determined by the use of the model are sufficiently accurate for either design or investigation, assuming the deflection theory values as a correct basis for comparison.

For values of H+Hw determined by the use of model, variations from those found by the conventional deflection theory, expressed in percentages, are

Case V od	0.49%	Case V of	0.097%
Case VII od	0.20%	Case VII of	-0.36%
Case VI 5d	0.55%	Case VI 5f	0.56%
Case XVI od	-0.32%	Case XVI of	-0.35%

These percentages of variation in H+Hw will be seen to be negligible, but as a matter of general interest should be contrasted with variations in values of H as determined by the model and those obtained by the conventional deflection theory. These variations in H as determined by the model from those determined by the conventional deflection theory, expressed in percentages, are

Case V od	-2.07%	Case V of	-6.3%
Case VII od	-1.73%	Case VII of	-5.68%
Case VI 5d	-0.35%	Case VI 5f	-0.355%
Case XVI od	-2.24%	Case XVI of	-2.4%

It will be seen that while the percentage of variation in H is relatively large, the variation in H+Hw is very small. This condition is an important factor in considering the model analysis, for it indicates that although H may vary quite markedly from the conventional deflection theory values, H+Hw will remain practically the same.

Variations in Hw, as measured from the model and as obtained from the deflection theory, have been discussed on page 18.

An inspection of Tables III and IV shows that in general the model gives larger deflections than the conventional deflection theory. This is influenced by three factors.

1. The effect of suspender stretch has been ignored in accordance with the usual practice in the calculations for the conventional deflection theory.

2. A wooden stiffening truss as shown and described was used. The Modulus of Elasticity of this wood was established by test of a piece cut from the same piece as the stiffening truss, but before the latter was brought down to its correct size. It is very possible that the Modulus of Elasticity of the whole piece forming the truss might have been smaller than that of the shorter piece, as this property is subject to much greater variation in wood than in metals. However, as average values for the Moment of Inertia of the stiffening truss are used in the deflection theory calculation, a small variation in the Modulus of Elasticity of the wood in the stiffening truss would not be an appreciable factor in considering comparative values.

3. Probably the most important factor is providing for the stretch of the backstay by increasing the length of center span cable by the effect of the H existing in the backstays before lengthening. Actually the effective H is not quite as large and hence the stretch proportionately smaller.

A comparison of results as indicated by moment and shear values show that these three above factors could have had no great effect. As an example of the small effect variations in deflections produce in the analysis, the following calculations are offered, which, however, represent an extreme case:

Consider Case VI 5f. Use all the values previously used, except arbitrarily change the value of the deflection from 6.20' to 5.70'. This gives

$H+Hw = 13,865,200\#$
$H = 3,296,200\#$
$n = 5.70'$
$M'\frac{\ell}{2}$ $637,692,927'\#$
$y = 145.3'$

$(H+Hw)n = 79,031,640$
$H(y) = 478,937,860$
$557,969,500$

$637,692,927$
$557,969,500$
$79,723,427'\#$

which shows a +8.9% variation from the value given by the deflection theory.

Thus even though decreasing the deflection, the moment is increased, contrary to any generally accepted theory, but illustrating well the effect of variation in deflection. Actually with such a large decrease in deflection there would be a corresponding change in H and therefore a smaller moment than that mentioned above.

These observations are made to show that in this method of model analysis reasonable variations in deflections are not of major

importance as affecting moments and shears except near the ends, where they become more marked in their effect, but model values there, are, of course, more accurate. Analytically, an inspection of the fundamental equation $M = M' - (H+Hw)n - H(y)$ reveals this, for the term $H(y)$ is always the largest of the negative ones on the right hand side.

An interesting fact observed from the action of the model was that short loads at the ends of the truss have a marked effect on the end shear. As soon as these loads move off towards the center, there is a very noticeable increase in the value of H for a relatively short movement of the loads. Short loads near the center of the truss can be moved quite a large amount with a very small percentage of increase in H. Graphs G17 and G18 illustrate this feature.

For a structure with suspended side spans a similar model for side spans can be made with the anchor bent support of cable towards the West end and with the East support pulley and lead to counterweight bucket a duplicate of the model shown. Working with these two models the horizontal components of the forces in the cable can be balanced by a step by step procedure.

No attempt has been made to analyse the effect of lateral forces. The difficulties presented in the various theories for analyzing these are not comparable to those encountered in the deflection theory for the design of the bridge proper.

A final summary of conclusions based on the results as previously interpreted is as follows.

For the type of suspension bridge considered, a model analysis, following the general method outlined, gives values for moments, shears, and cable stresses that are well within any required degree of accuracy for design or investigation, using the conventional deflection theory of suspension bridge design, as a normal basis of comparison. Deflection values as determined from the model are fairly accurate but not precise.

That with the usual ratios existing between total dead load and total live load on bridges of this type, values of H+Hw, as determined by the model, show variations compared to those determined by the conventional deflection theory that are negligible. The values of H show a larger variation but that the effect of this variation practically disappears when these values are used in the basic formula for determining M.

That model analysis, by establishing properly interpreted data, can be used as a convenient method of giving physical significance to certain factors in the conventional deflection theory which heretofore have had only a mathematical conception.

GENERAL OBSERVATIONS ON THE CONSTRUCTION AND USE OF MODELS.

The model described in this paper was built using only commercially available material that could be purchased at a modest cost. The use of wood in the stiffening truss may appear at first glance to be out of place in such a model, but this seems to be due primarily to the mental picturing of a model as necessarily a physical reproduction of the field bridge. Wood was decided upon after preliminary experiments in which both wood and steel were tried, and which showed only minor differences in the results obtained for the more important factors. Wood was finally selected on account of its adaptability to the details required and to the conditions under which the model had to be built. A piece of selected wood was used and the Modulus of Elasticity of a short piece cut from same was carefully determined. Further study and trial might show that there are other materials that could be used to better advantage in satisfying the various features that have to be taken into account. Where stiffening trusses of varying Moment of Inertia are to be given their model equivalent, wood seems more easily adapted to the required conditions than any material that has been suggested.

From the experience developed in the construction and use of this model, a number of improvements that would add to its use and efficiency appear to be practicable, but the most of these would involve considerable additional expense and would require the use of specially designed parts. This expense, while it would be considered large for an individual to incur for purely experimental purposes, would not be great when measured by the usual standards of expense incurred in planned research work.

Some of the improvements and developments for further model study as suggested by this experiment are:

1. A wire exactly proportioned for loads can be selected for model suspenders, permitting the suspender stretch to have its proper effect in the elastic curve of the stiffening truss.

2. A special "floating" pulley can be designed for the East end supporting pulley with a horizontal lead to the counterweight bucket that will permit the H or Hw to be registered directly by load in the counterweight bucket, thus eliminating the work of transferring the cable tension into H or Hw.

3. A model can be built along the lines illustrated covering both the center and side spans (suspended or unsuspended), that will register H or Hw directly and give direct effect to backstay stretch or suspended side span action. Also this model can be made to apply to cases with stiffening trusses stopping at the towers or with stiffening trusses continuous by the towers.

4. A model can be built along the lines illustrated that will give effect to the tower bending and permit the application or determination of the forces set up by the same while showing their effect on the structure as a whole. This type of model can also be developed to show the effect of tower ties on continuous suspension spans.

5. A twin model can be built along the lines illustrated that will combine all of the features mentioned and will in addition show and register the effects of lateral forces on cables and horizontal wind truss.

6. All models of the type mentioned, after being used for the investigation of a particular structure, can be readily disconnected and practically all the parts except cable wire, suspenders, and stiffening truss used in another model "set up". In other words, the greater part of the materials, after once being properly designed and collected, could be considered as standard laboratory equipment for general use in this type of model analysis.

Some of these suggestions in the construction and use of such models are made primarily from the viewpoint of a purely scientific research and study organized to explore certain fields in this subject, where mathematical analyses seem deficient or where the assumptions upon which they are applied seem to require proper verification. A study of this nature, if it merely confirmed all present theories, would not from this viewpoint be wasted, but its chances of producing results of a practical or commercial value are those accompanying any well organized research and the possibilities of the field to be explored, few of which can be predicted in advance. Such investigations have to meet the objections of the school of thought that can see no object in such a research unless practical or commercial results are assured in advance, and the school that assumes all knowledge in their particular field to be complete and final.